Algebra Lecture 9

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Topics

Today

1. Systems of Linear Equations

What is a System of Linear Equations?

It is a set of Linear Equations.

A Set of n Linear Equations

$$\begin{cases} y_1 = m_1 x + b_1 \\ y_2 = m_2 x + b_2 \\ \vdots \\ y_n = m_n x + b_n \end{cases}$$

Here, we will consider sets of two linear equations.

$$\begin{cases} y_1 = m_1 x + b_1 \\ y_2 = m_2 x + b_2 \end{cases}$$

A System of Linear Equations is said to be:

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Consistent if it has at least one solution

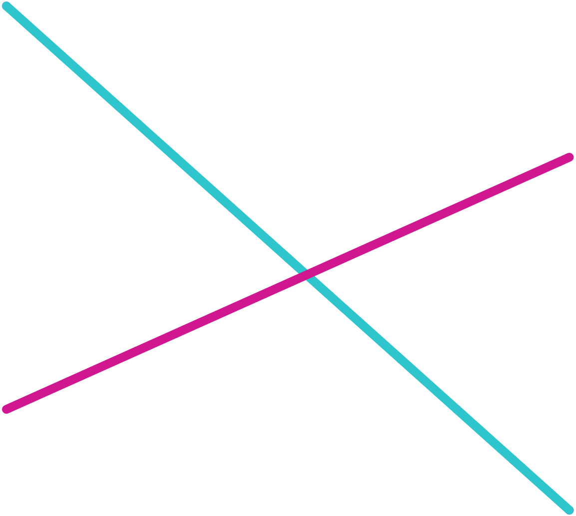
A System of Linear Equations is said to be:

Independent if it has exactly one solution (dependent otherwise).

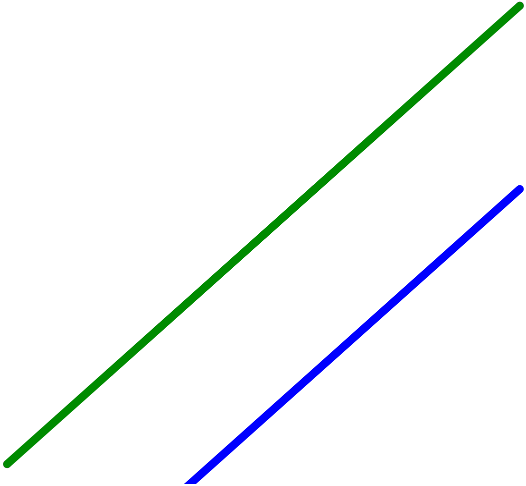
Consistent if it has at least one solution (inconsistent otherwise).

Recall the possible ways in which two lines can be drawn in the plane $\mathbb{R}^2$.

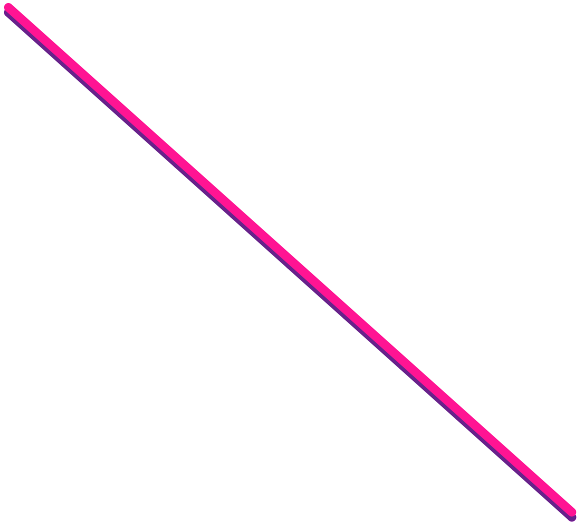
Independent & Consistent



Inconsistent & Independent



Dependent & Consistent



A solution to a system of linear equations is a set of points that satisfy all linear equations in the system.

As with solutions for a single linear equation, there are only three possibilities for the solutions to a System of two Linear Equations.

Three Possibilities for a System of Two Linear Equations.

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1. There is **One** solution.

Three Possibilities for a System of Two Linear Equations.

1. There is **One** solution.
2. There are **Zero** solutions.

Three Possibilities for a System of Two Linear Equations.

1. There is **One** solution.
2. There are **Zero** solutions.
3. There are **Infinitely** many solutions.

Examples

Consider the following linear system

$$\clubsuit = \begin{cases} -6x + 7y = 29 \\ 3x + y = 16 \end{cases}$$

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Is the point $(5, 1)$ a solution of the linear system $\clubsuit$?

Substitute the Point $(5, 1)$ to Check

$$-6x + 7y = 29$$

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$$-6(5) + 7(1) \stackrel{?}{=} 29$$

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$$-23 \stackrel{?}{=} 29$$

Substitute the Point $(5, 1)$ to Check

$$-6x + 7y = 29$$

$$-6(5) + 7(1) \stackrel{?}{=} 29$$

$$-30 + 7 \stackrel{?}{=} 29$$

$$-23 \stackrel{?}{=} 29$$

NO!

The point $(5, 1)$ is not a solution to the linear system $\clubsuit$ because it fails to satisfy both equations.

$$\clubsuit = \begin{cases} -6x + 7y = 29 \\ 3x + y = 16 \end{cases}$$

Consider the following linear system

$$\heartsuit = \begin{cases} x - y = 4 \\ 2x - 2y = 4 \end{cases}$$

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How many solutions are there for the linear system $\heartsuit$?

Solve for the Solutions Algebraically

$$-2(x - y) = -2(4)$$

$$2x - 2y = 4$$

Solve for the Solutions Algebraically

$$-2x + 2y = -8$$

$$2x - 2y = 4$$

$$\overline{}$$

$$0 = -4$$

Solve for the Solutions Algebraically

$$
\begin{array}{r}
-2x + 2y = -8 \\
2x - 2y = 4 \\
\hline
0 = -4
\end{array}
$$

Not Possible!

Solve for the Solutions Algebraically

$$\begin{array}{r} -2x + 2y = -8 \\ 2x - 2y = 4 \\ \hline 0 = -4 \end{array}$$

Not Possible! There are **no solutions** for the linear system ♡.

Solve for the Solutions Algebraically

$$-2x + 2y = -8$$

$$2x - 2y = 4$$

$$0 = -4$$

Not Possible! There are **no solutions** for the linear system $\heartsuit$.

$\varnothing$ denotes the empty set $\{\}$.

The linear system $\heartsuit$ is inconsistent.

$$\heartsuit = \begin{cases} x - y = 4 \\ 2x - 2y = 4 \end{cases}$$

Consider the following linear system

$$\blacklozenge = \begin{cases} 2x + 4y = 2 \\ -x - 2y = -1 \end{cases}$$

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How many solutions are there for the linear system $\blacklozenge$?

Solve for the Solutions Algebraically

$$2x + 4y = 2$$

$$2(-x - 2y) = 2(-1)$$

Solve for the Solutions Algebraically

$$
\begin{array}{r}
2x + 4y = 2 \\
-2x - 4y = -2 \\
\hline
0 = 0
\end{array}
$$

This is always true!

There are infinitely many solutions to the linear system $\blacklozenge$.

$$\blacklozenge = \begin{cases} 2x + 4y = 2 \\ -x - 2y = -1 \end{cases}$$

The solutions are $\{(x, y) | -x - 2y = -1\}$.

The linear system $\blacklozenge$ is consistent and dependent.

$$\blacklozenge = \begin{cases} 2x + 4y = 2 \\ -x - 2y = -1 \end{cases}$$

The solutions are $\{(x, y) | -x - 2y = -1\}$.

Solve the following linear system

$$\spadesuit = \begin{cases} \frac{1}{2}x - \frac{1}{3}y = \frac{5}{6} \\ \frac{1}{5}x - \frac{1}{4}y = \frac{15}{10} \end{cases}$$

Word Problems

The **perimeter** of a rectangle is 42 feet.

The length is seven feet more than the width.

Find the **dimensions** of the rectangle.

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Find the **dimensions** of the rectangle.

System of Linear Equations for Perimeter and Length

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$$\begin{cases} 2L + 2W = 42 \\ \end{cases}$$

System of Linear Equations for Perimeter and Length

$$\begin{cases} 2L + 2W = 42 \\ L = W + 7 \end{cases}$$

$$2(W + 7) + 2W = 42$$

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$$2W + 14 + 2W = 42$$

$$2(W + 7) + 2W = 42$$

$$2W + 14 + 2W = 42$$

$$4W = 28$$

$$2(W + 7) + 2W = 42$$

$$2W + 14 + 2W = 42$$

$$4W = 28$$

$$\boxed{W = 7}$$

$$2(W + 7) + 2W = 42$$

$$2W + 14 + 2W = 42$$

$$4W = 28$$

$$\boxed{W = 7} \rightarrow \boxed{L = 7 + 7 = 14}$$

The **sum** of two numbers is 13, and their difference is 5.

Find the numbers.

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Find the numbers.

System of Linear Equations for Sum and Difference

System of Linear Equations for Sum and Difference

$$\begin{cases} x + y = 13 \\ \end{cases}$$

System of Linear Equations for Sum and Difference

$$\begin{cases} x + y = 13 \\ x - y = 5 \end{cases}$$

Next Time

Systems of Linear Inequalities