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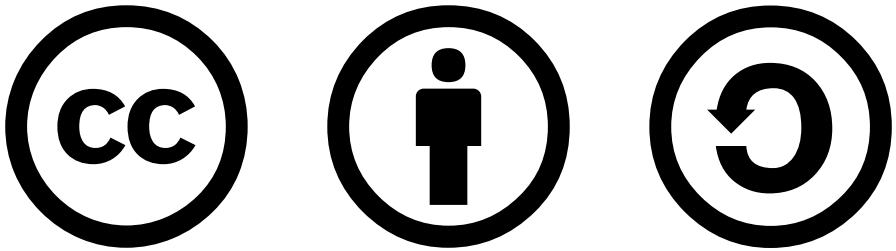


Algebra Lecture 7

Crista Moreno

December 4, 2016

Today

1. Solving Linear Equations

Properties of Arithmetic

$$ab + ac = a(b + c)$$

$$a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{a - b}{c - d} = \frac{b - a}{d - c}$$

Properties of Arithmetic Continued

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab+ac}{a} = b + c, a \neq 0$$

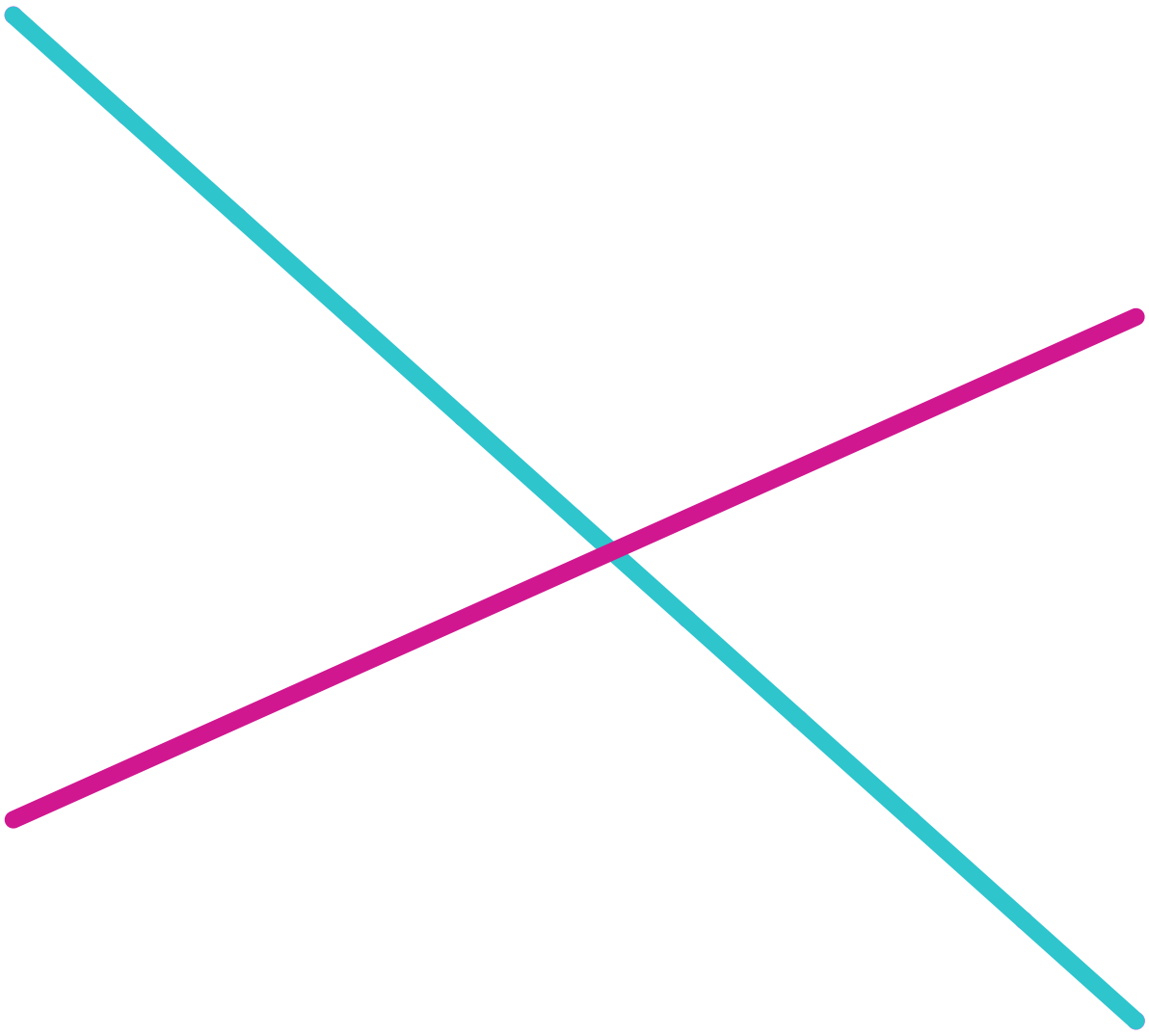
$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$$

What is a Linear Equation?

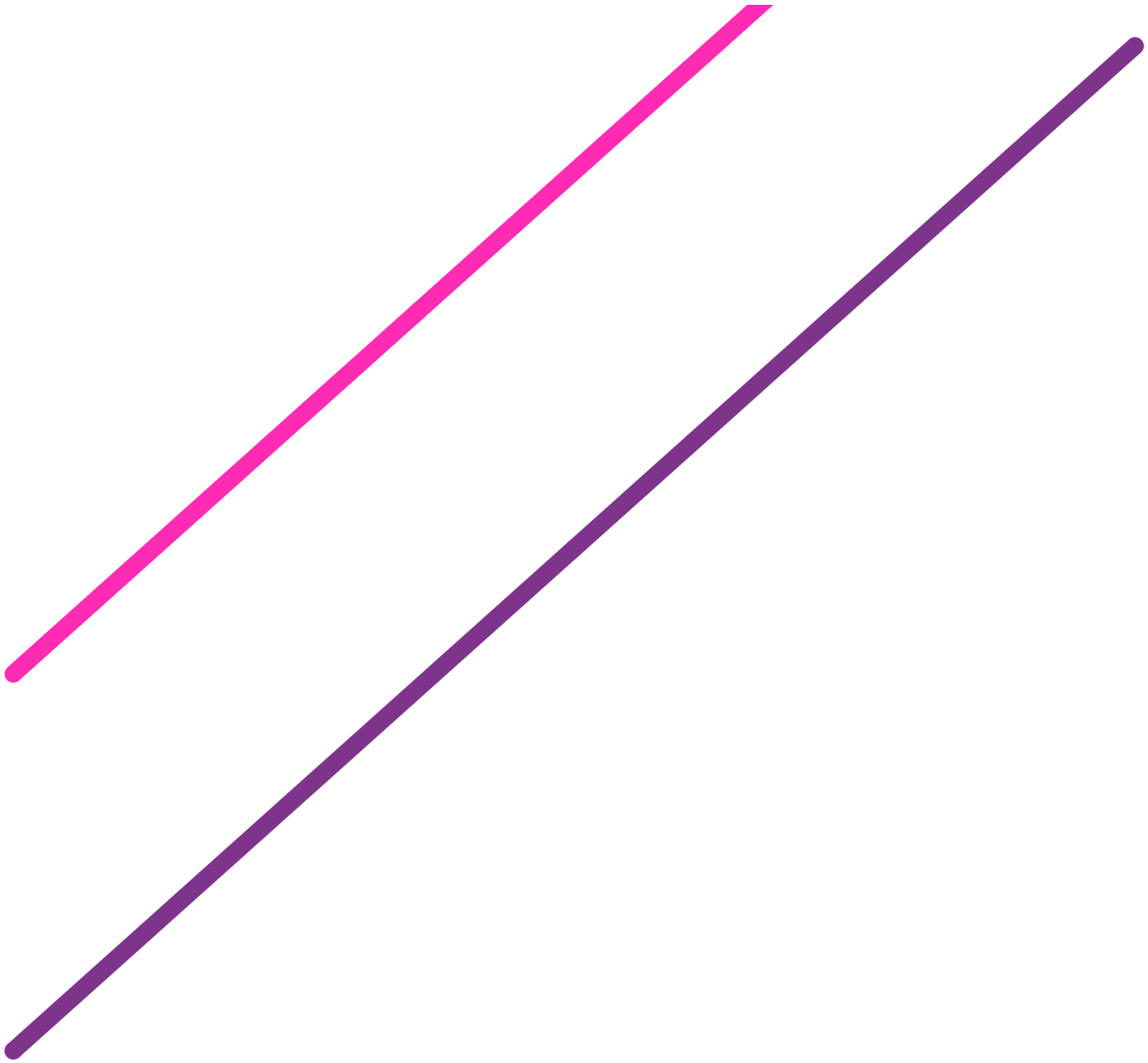
It is an equation involving lines.

What are the possible things that can happen with two lines in the plane $\mathbb{R}^2$?

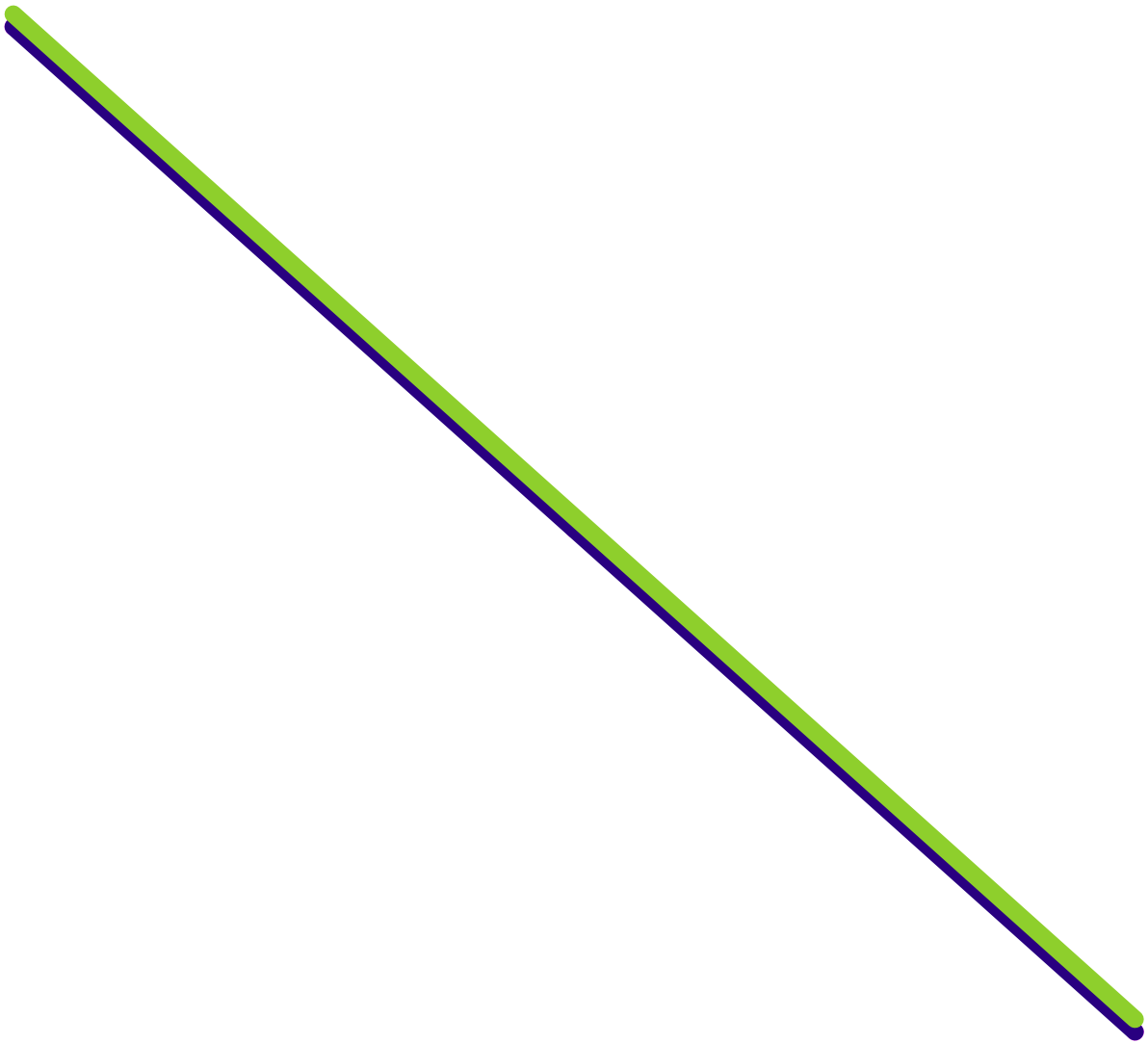
One Possibility



Another Possibility



Yet Another Possibility



Three Possibilities for Two Lines to interact.

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1. They touch **Once**.

Three Possibilities for Two Lines to interact.

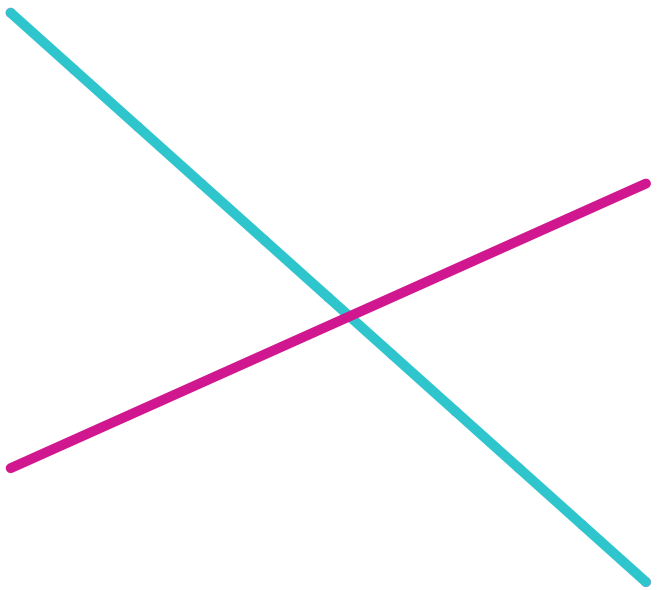
1. They touch **Once**.
2. They **Never** touch.

Three Possibilities for Two Lines to interact.

1. They touch **Once**.
2. They **Never** touch.
3. They **Always** touch.

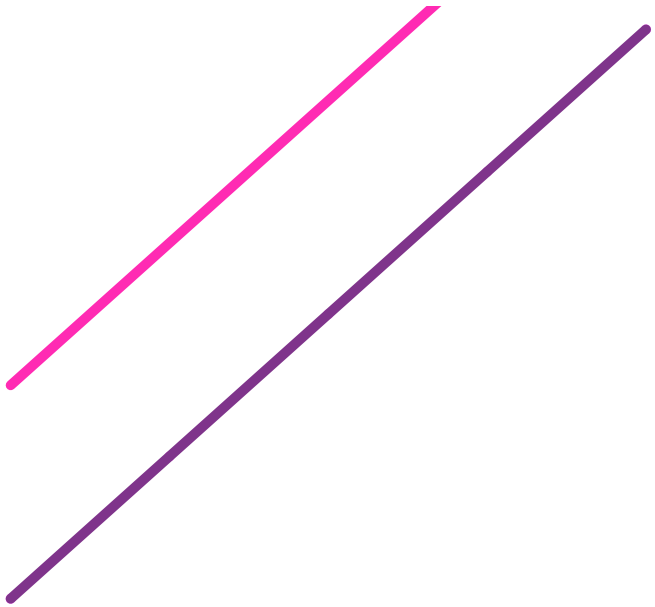
When do the two lines touch only **Once**?

When the two lines have different slopes.



When do the two lines **Never** touch?

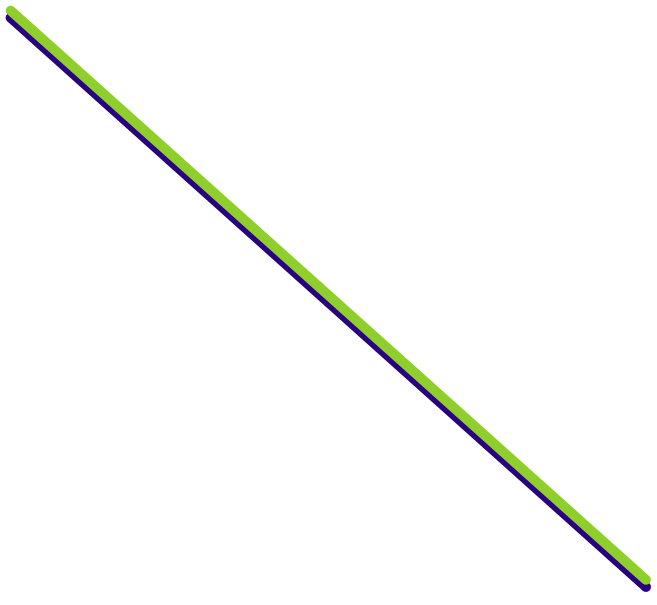
When the two lines have the same slope.



Two lines having the same slope means that they are (parallel).

When do the two lines **Always** touch?

When the two lines are the same.



The **solutions** to a Linear Equation represent where the two lines touch, and the **number** of solutions represent the number of times they touch. The only possible number of times is $0, 1,$ **or** ∞.

Determine the number of solutions for the following linear equation

$$\frac{-1}{3}(4-x)+\frac{2}{5}(x+1)=\frac{-1}{5}$$

$$\frac{-1}{3}(4-x)+\frac{2}{5}(x+1)=\frac{-1}{5}$$

$$\frac{-1}{3}(4-x)+\frac{2}{5}(x+1)=\frac{-1}{5}$$

$$y=\frac{-1}{3}(4-x)+\frac{2}{5}(x+1)$$

$$\frac{-1}{3}(4-x)+\frac{2}{5}(x+1)=\frac{-1}{5}$$

$$y=\frac{-1}{3}(4-x)+\frac{2}{5}(x+1)$$

$$y=\frac{-4}{3}+\frac{1}{3}x+\frac{2}{5}x+\frac{2}{5}$$

$$\frac{-1}{3}(4-x)+\frac{2}{5}(x+1)=\frac{-1}{5}$$

$$y=\frac{-1}{3}(4-x)+\frac{2}{5}(x+1)$$

$$y=\frac{-4}{3}+\frac{1}{3}x+\frac{2}{5}x+\frac{2}{5}$$

$$y=\frac{-4}{3}+\frac{11}{15}x+\frac{2}{5}$$

$$\frac{-1}{3}(4-x)+\frac{2}{5}(x+1)=\frac{-1}{5}$$

$$y=\frac{-1}{3}(4-x)+\frac{2}{5}(x+1)$$

$$y=\frac{-4}{3}+\frac{1}{3}x+\frac{2}{5}x+\frac{2}{5}$$

$$y=\frac{-4}{3}+\frac{11}{15}x+\frac{2}{5}$$

$$\boxed{y=\frac{11}{15}x+\frac{-14}{15}}$$

$$\frac{-1}{3}(4-x)+\frac{2}{5}(x+1)=\frac{-1}{5}$$

$$y=\frac{-1}{3}(4-x)+\frac{2}{5}(x+1)$$

$$y=\frac{-4}{3}+\frac{1}{3}x+\frac{2}{5}x+\frac{2}{5}$$

$$y=\frac{-4}{3}+\frac{11}{15}x+\frac{2}{5}$$

$$\boxed{y=\frac{11}{15}x+\frac{-14}{15}}$$

$$\boxed{y=\frac{-1}{5}}$$

We observe that the slopes of the two lines

$$y = \frac{11}{15}x + \frac{-14}{15} \text{ and } y = \frac{-1}{5}$$

are different, so we expect the **number of solutions** to be **one**.

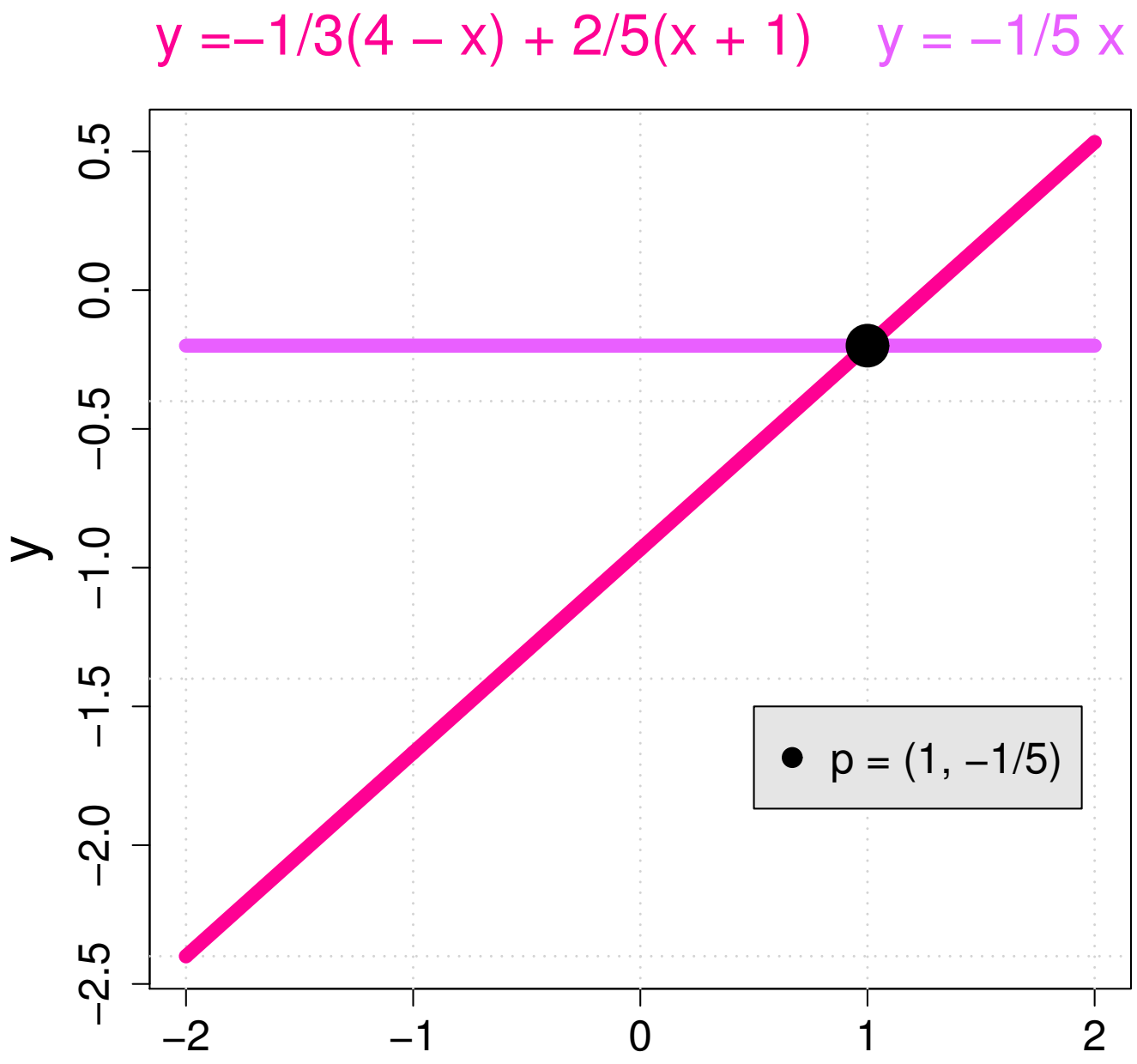
We observe that the slopes of the two lines

$$y = \frac{11}{15}x + \frac{-14}{15} \text{ and } y = \frac{-1}{5}$$

are different, so we expect the **number of solutions** to be **one**.

The slopes are $\frac{11}{15}$ and 0.

Graph



Is $x = 1$ a Solution?

$$\frac{-1}{3}(4 - x) + \frac{2}{5}(x + 1) = \frac{-1}{5}$$

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$$\frac{-1}{3}(4 - x) + \frac{2}{5}(x + 1) = \frac{-1}{5}$$

$$\frac{-1}{3}(4 - (1)) + \frac{2}{5}((1) + 1) = \frac{-1}{5}$$

Is $x = 1$ a Solution?

$$\frac{-1}{3}(4 - x) + \frac{2}{5}(x + 1) = \frac{-1}{5}$$

$$\frac{-1}{3}(4 - (1)) + \frac{2}{5}((1) + 1) = \frac{-1}{5}$$

$$\frac{-1}{3}(3) + \frac{2}{5}(2) = \frac{-1}{5}$$

Is $x = 1$ a Solution?

$$\frac{-1}{3}(4 - x) + \frac{2}{5}(x + 1) = \frac{-1}{5}$$

$$\frac{-1}{3}(4 - (1)) + \frac{2}{5}((1) + 1) = \frac{-1}{5}$$

$$\frac{-1}{3}(3) + \frac{2}{5}(2) = \frac{-1}{5}$$

$$\frac{-3}{3} + \frac{4}{5} = \frac{-1}{5}$$

Is $x = 1$ a Solution?

$$\frac{-3}{3} + \frac{4}{5} = \frac{-1}{5}$$

Is $x = 1$ a Solution?

$$\frac{-3}{3} + \frac{4}{5} = \frac{-1}{5}$$

$$\frac{-3}{3} = \frac{-1}{5} - \frac{4}{5}$$

Is $x = 1$ a Solution?

$$\frac{-3}{3} + \frac{4}{5} = \frac{-1}{5}$$

$$\frac{-3}{3} = \frac{-1}{5} - \frac{4}{5}$$

$$\frac{-3}{3} = \frac{-5}{5}$$

Is $x = 1$ a Solution?

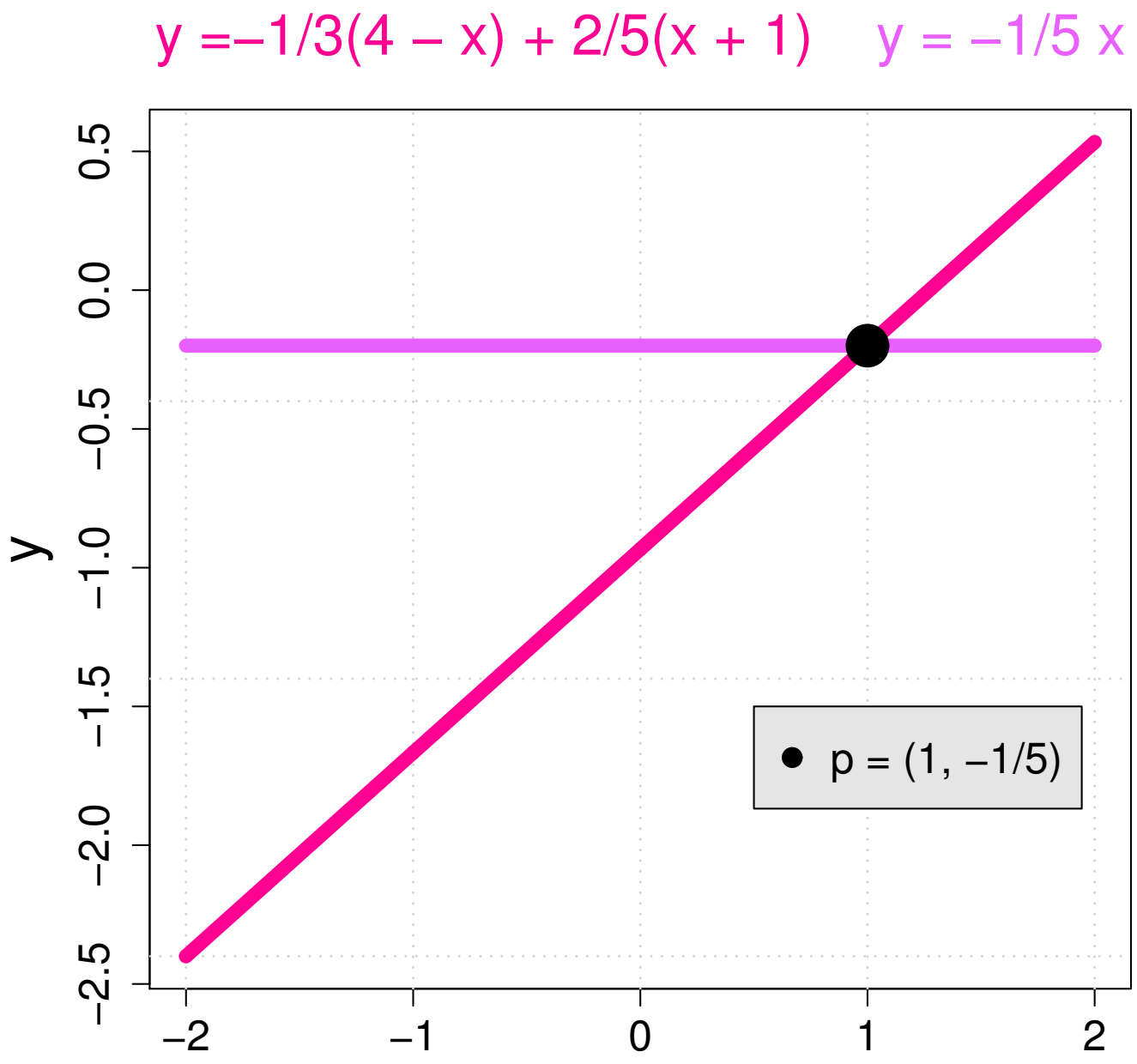
$$\frac{-3}{3} + \frac{4}{5} = \frac{-1}{5}$$

$$\frac{-3}{3} = \frac{-1}{5} - \frac{4}{5}$$

$$\frac{-3}{3} = \frac{-5}{5}$$

$$\boxed{-1 = -1} \qquad \textit{Yes!}$$

Graph



Determine the number of solutions for the following linear equation

$$2x - 5 = 6 - \frac{5}{2}x$$

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$$2x - 5 = 6 - \frac{5}{2}x$$

$$\boxed{y = 2x - 5}$$

$$2x - 5 = 6 - \frac{5}{2}x$$

$$y = 2x - 5$$

$$y = 6 - \frac{5}{2}x$$

Again we observe that the slopes of the two lines

$y = 2x - 5$ and $y = 6 - \frac{5}{2}x$

are different, so we expect the **number of solutions** to be **one**.

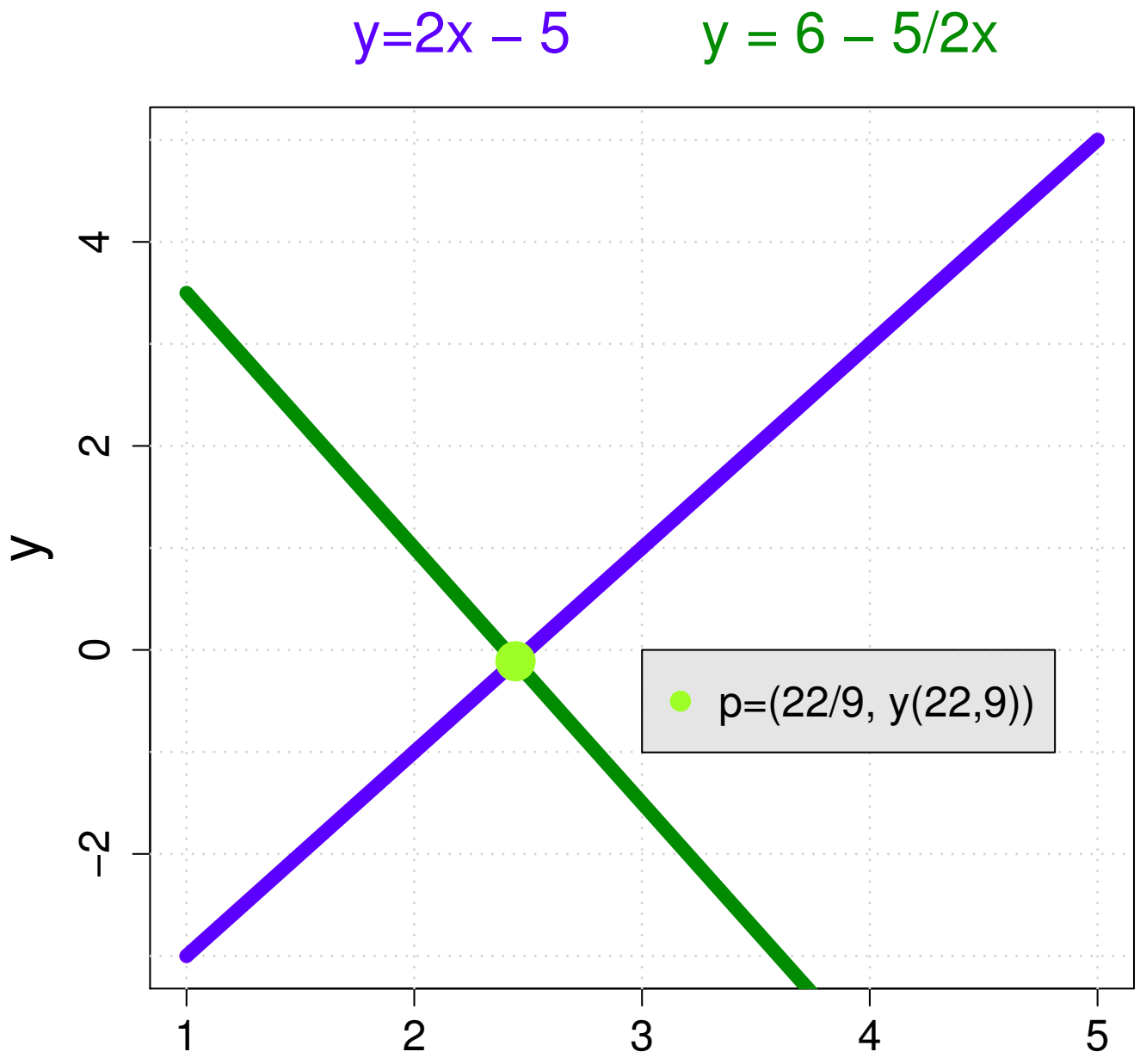
Again we observe that the slopes of the two lines

$y = 2x - 5$ and $y = 6 - \frac{5}{2}x$

are different, so we expect the **number of solutions** to be **one**.

The slopes are 2 and $-\frac{5}{2}$.

Graph



Find the Solution Algebraically

$$2x - 5 = 6 - \frac{5}{2}x$$

Find the Solution Algebraically

$$2x - 5 = 6 - \frac{5}{2}x$$

$$2\left(2x - 5\right) = 2\left(6 - \frac{5}{2}x\right)$$

Find the Solution Algebraically

$$2x - 5 = 6 - \frac{5}{2}x$$

$$2\left(2x - 5\right) = 2\left(6 - \frac{5}{2}x\right)$$

$$4x - 10 = 12 - 5x$$

Find the Solution Algebraically

$$2x - 5 = 6 - \frac{5}{2}x$$

$$2\,(2x - 5) = 2\left(6 - \frac{5}{2}x\right)$$

$$4x - 10 = 12 - 5x$$

$$4x = 12 + 10 - 5x$$

Find the Solution Algebraically

$$4x = 12 + 10 - 5x$$

Find the Solution Algebraically

$$4x = 12 + 10 - 5x$$

$$4x = 22 - 5x$$

Find the Solution Algebraically

$$4x = 12 + 10 - 5x$$

$$4x = 22 - 5x$$

$$4x + 5x = 22$$

Find the Solution Algebraically

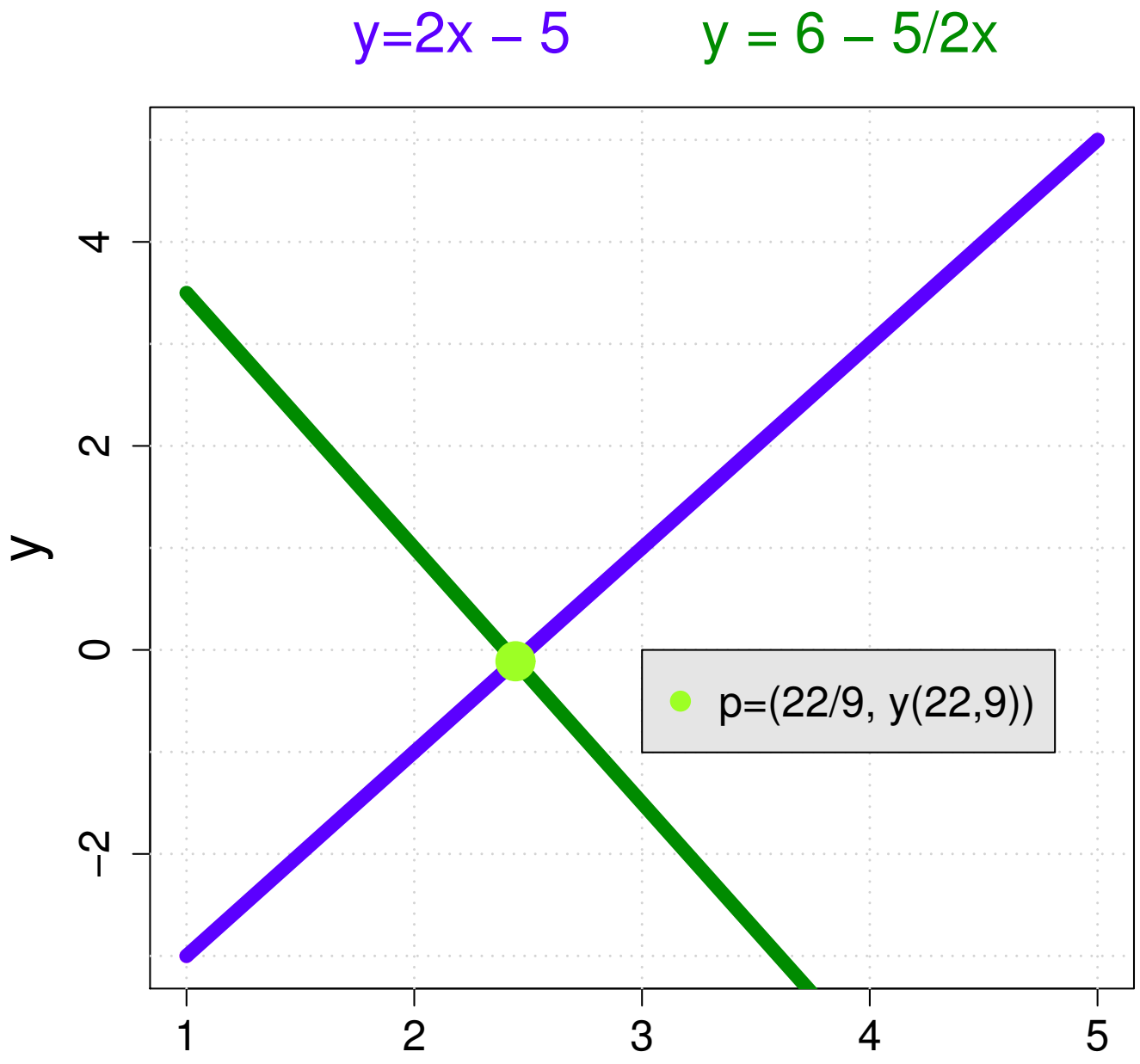
$$4x = 12 + 10 - 5x$$

$$4x = 22 - 5x$$

$$4x + 5x = 22$$

$$9x = 22 \rightarrow \boxed{x = \frac{22}{9} \sim 2.45}$$

Graph



Determine the number of solutions for the following linear equation

$$5(x-2)+4=5x+7$$

$$5(x-2)+4=5x+7$$

$$5(x-2)+4=5x+7$$

$$y=5x-10+4$$

$$5(x-2)+4=5x+7$$

$$y=5x-10+4$$

$$\boxed{y=5x-6}$$

$$5(x-2)+4 = 5x+7$$

$$y = 5x - 10 + 4$$

$$\boxed{y = 5x - 6}$$

$$\boxed{y = 5x + 7}$$

We observe that the slopes of the two lines

$y = 5x - 6$ and $y = 5x + 7$

are **equal**, but they are **different lines**. So we expect that the linear equation has **zero** solutions.

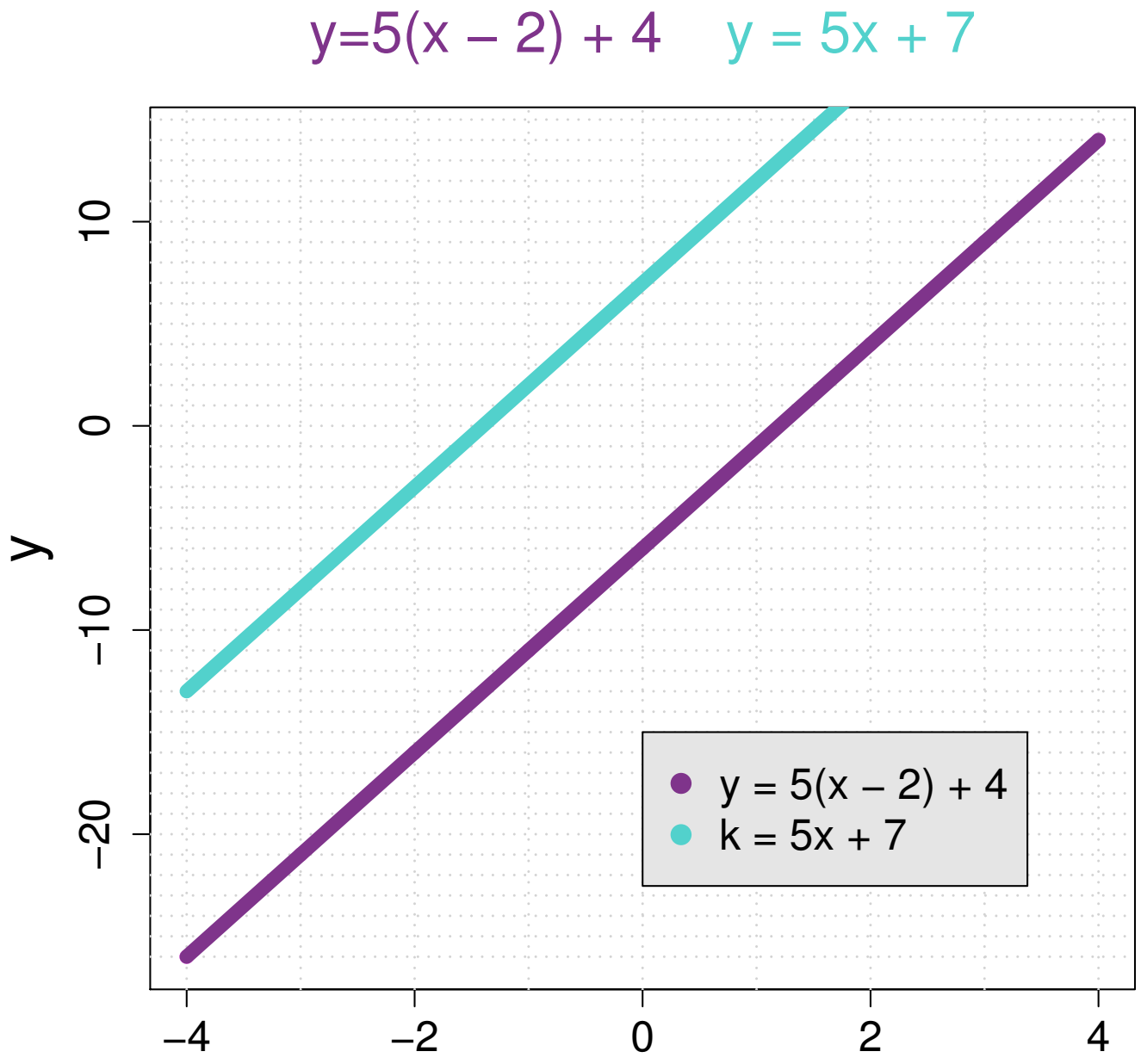
We observe that the slopes of the two lines

$y = 5x - 6$ and $y = 5x + 7$

are **equal**, but they are **different lines**. So we expect that the linear equation has **zero** solutions.

The slopes are 5 and 5.

Graph



Solve for Solutions Algebraically

$$5(x-2)+4=5x+7$$

Solve for Solutions Algebraically

$$5(x-2)+4=5x+7$$

$$5x-10+4=5x+7$$

Solve for Solutions Algebraically

$$5(x-2)+4=5x+7$$

$$5x-10+4=5x+7$$

$$5x-6=5x+7$$

Solve for Solutions Algebraically

$$5(x-2)+4=5x+7$$

$$5x-10+4=5x+7$$

$$5x-6=5x+7$$

$$5x=5x+7+6$$

Solve for Solutions Algebraically

$$5x = 5x + 13$$

Solve for Solutions Algebraically

$$5x = 5x + 13$$

$$5x - 5x = 13$$

Solve for Solutions Algebraically

$$5x = 5x + 13$$

$$5x - 5x = 13$$

$$0 = 13$$

Solve for Solutions Algebraically

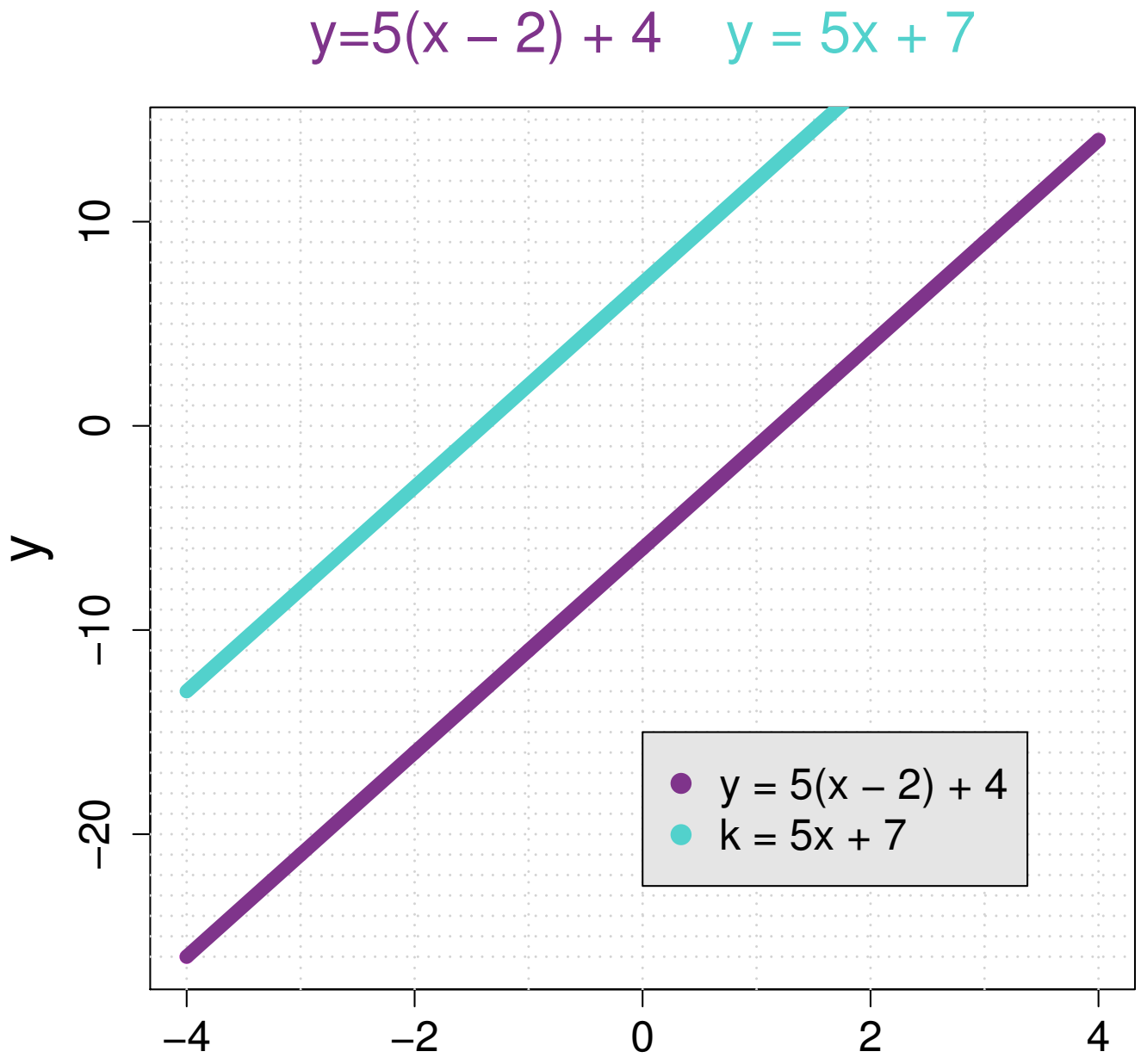
$$5x = 5x + 13$$

$$5x - 5x = 13$$

$$0 = 13 \rightarrow$$

Not Possible! No Solution!

Graph



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