Algebra Lecture 5

Crista Moreno

December 19, 2016

Topics

Last Time

1. Relations
2. Domains
3. Ranges
4. Functions
5. Vertical Line Test

Topics

Today

1. Linear Functions
2. Slope

Recall Definition of a Function

A relation in which each x-coordinate is matched with only one y-coordinate is said to describe y as a function of x.

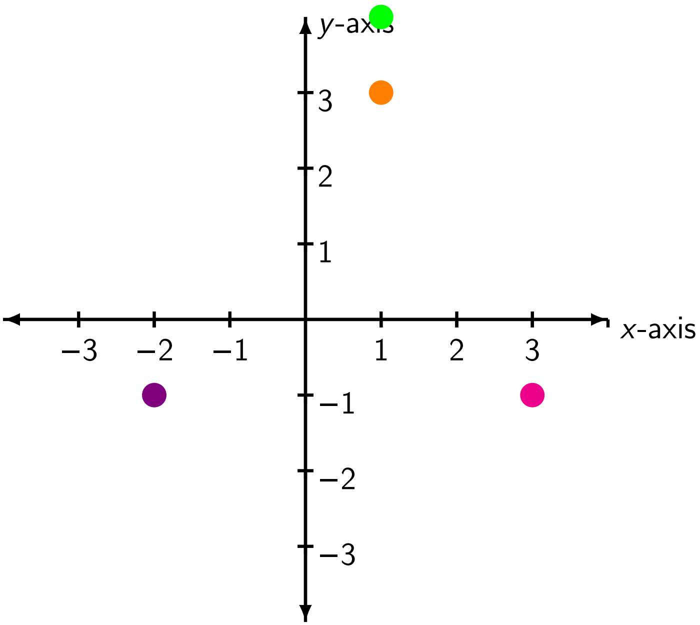
Recall Definition of a Function

Which of the following relations describe y as a function of x?

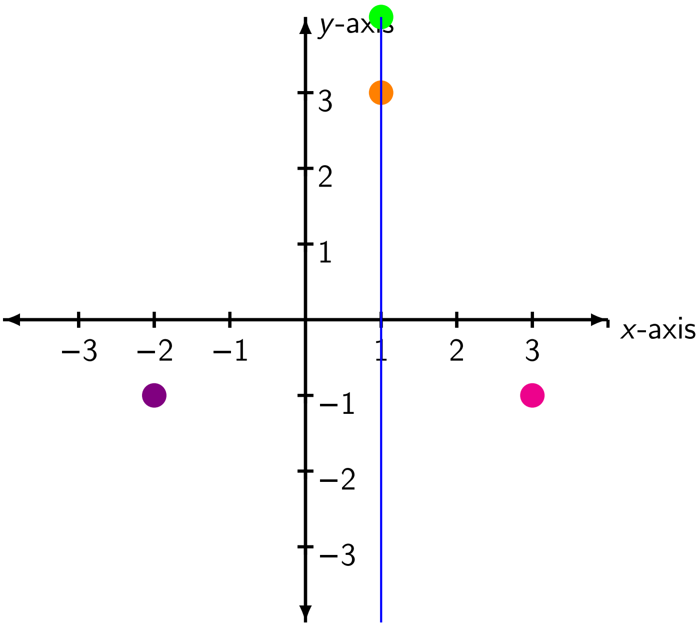
$$R_1 = \{(-2, 1), (1, 3), (1, 4), (3, -1)\}$$

$$R_2 = \{(-2, 1), (1, 3), (2, 3), (3, -1)\}$$

Plot $R_1 = \{(-2,1),(1,3),(1,4),(3,-1)\}$ in $\mathbb{R}^2$



The relation R_1 Fails the Vertical Line Test!



The Vertical Line Test: A set of points in the plane represents y as a function of x if and only if no two points lie on the same vertical line.

Linear Functions

What defines a line?

What Defines a Line?

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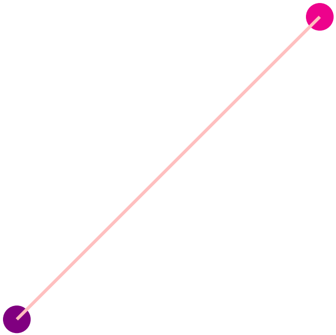

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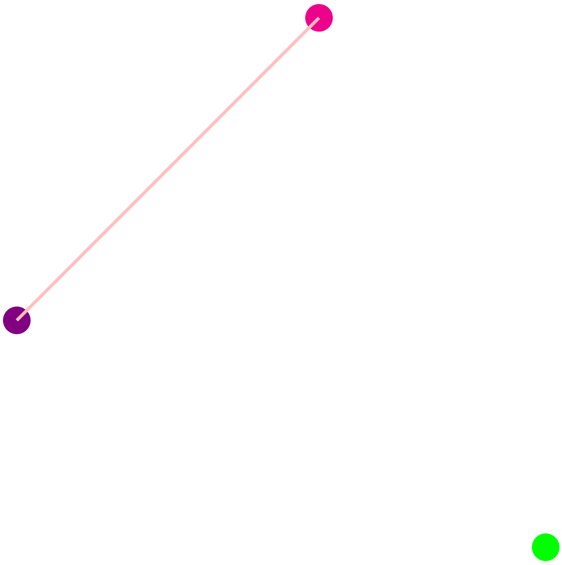

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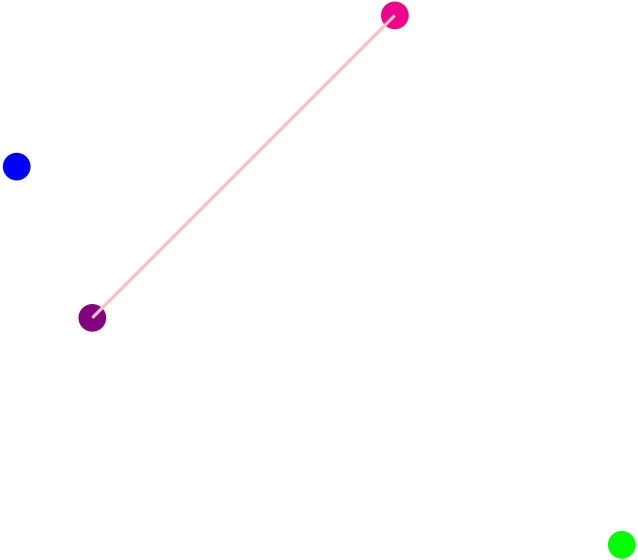

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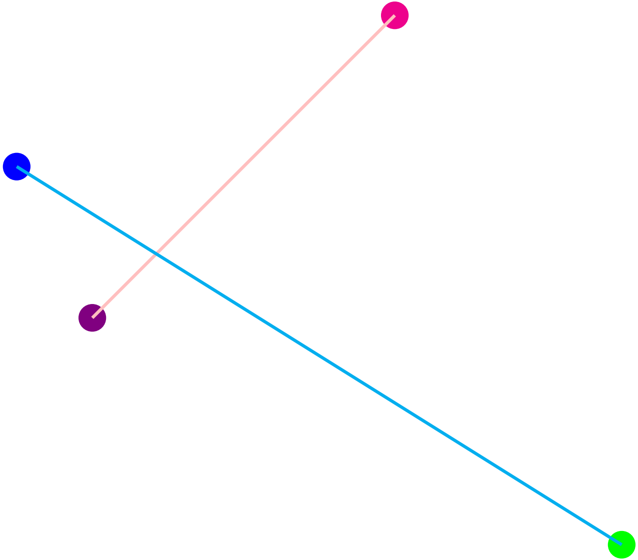

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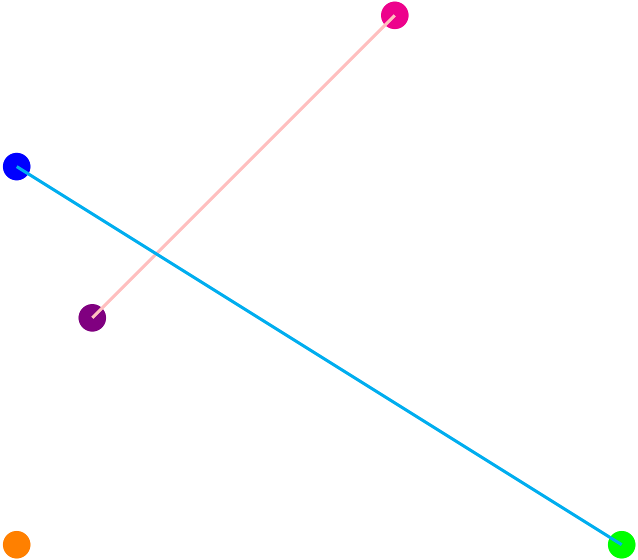

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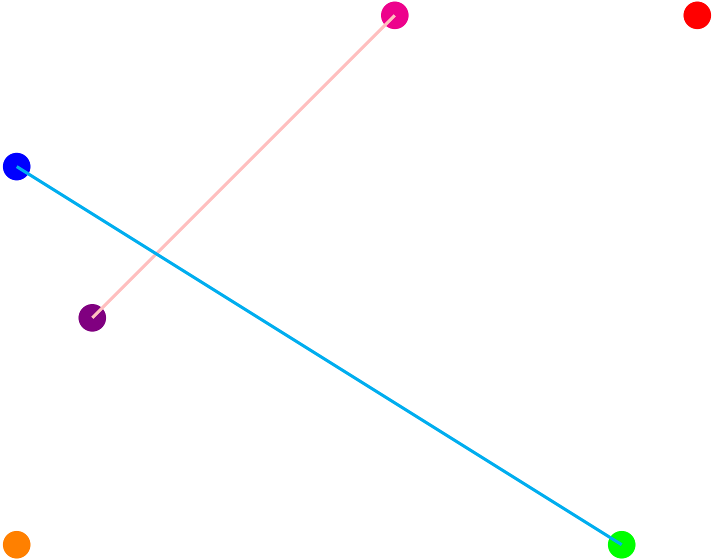

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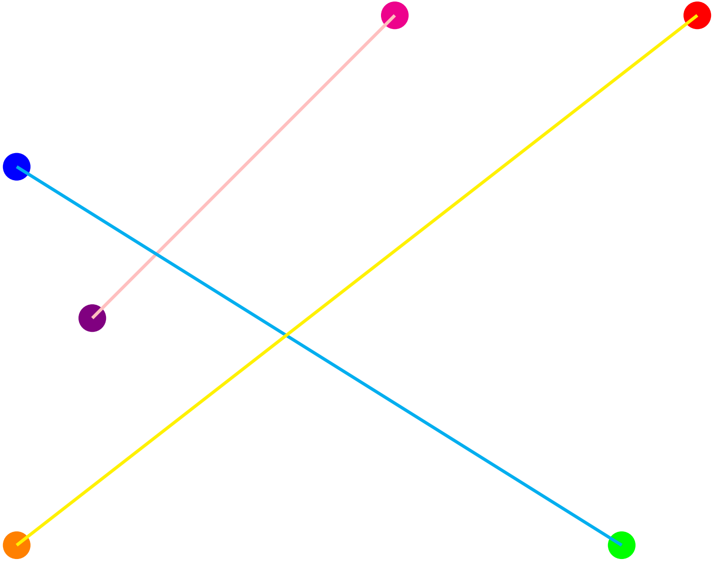

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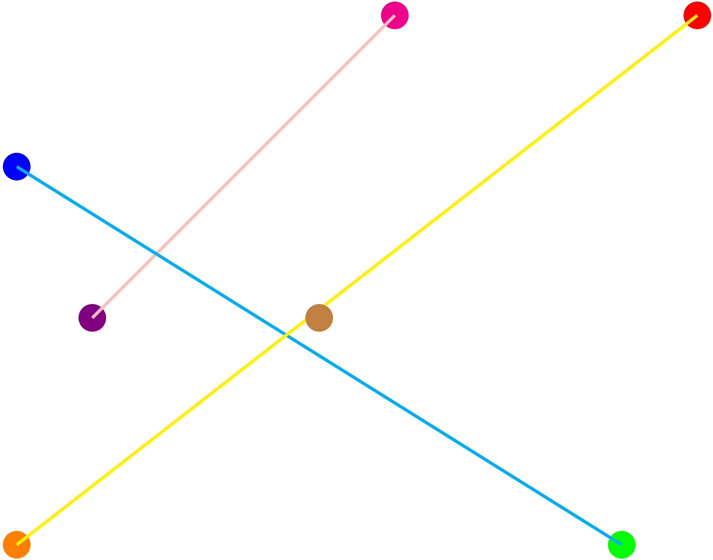

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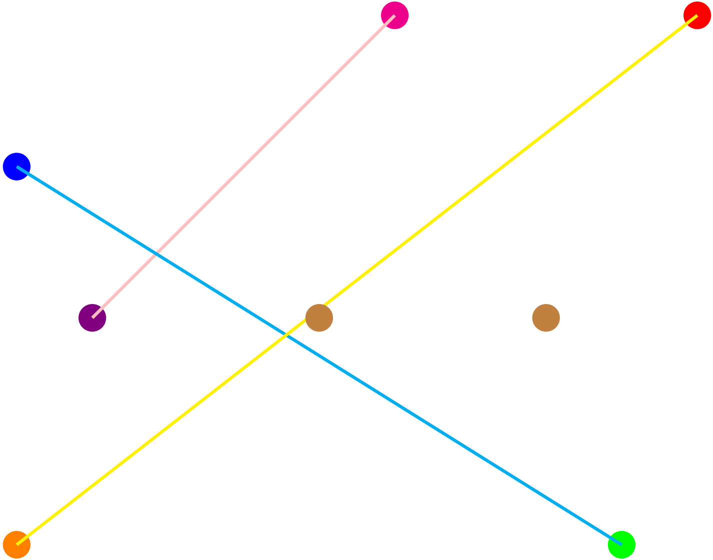

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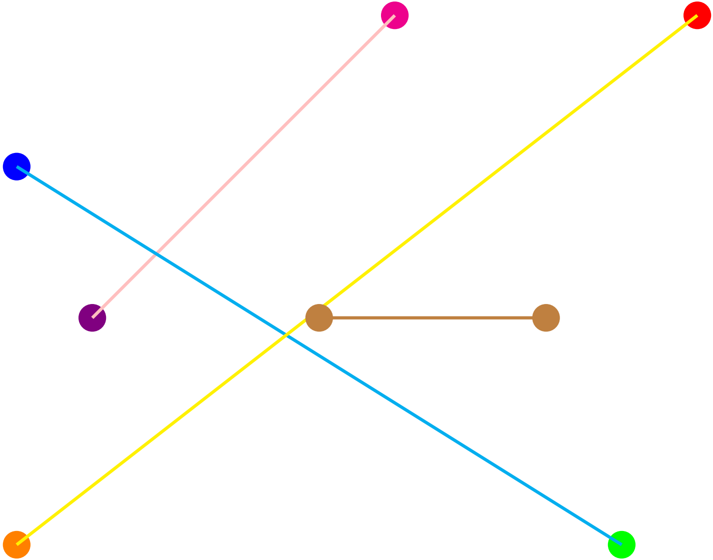
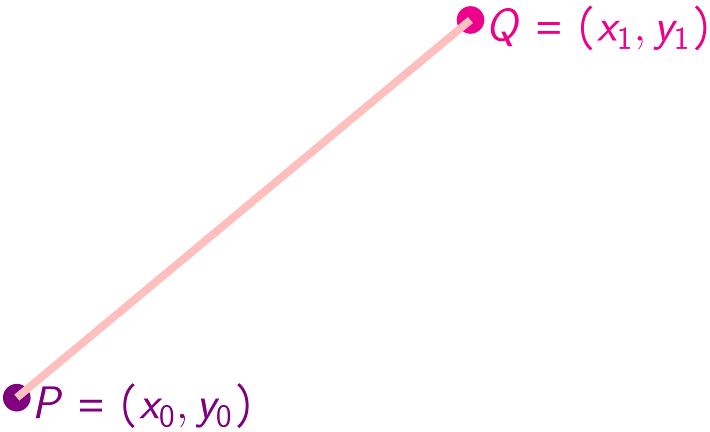


Two points $P = (x_0, y_0)$ and $Q = (x_1, y_1)$

define a line.

What is the Slope of a Line?



The **slope** (or ‘steepness’) m of the line containing the points $P = (x_0, y_0)$ and $Q = (x_1, y_1)$ is

$$\boxed{m = \frac{y_1 - y_0}{x_1 - x_0}}$$

provided $x_1 \neq x_0$.

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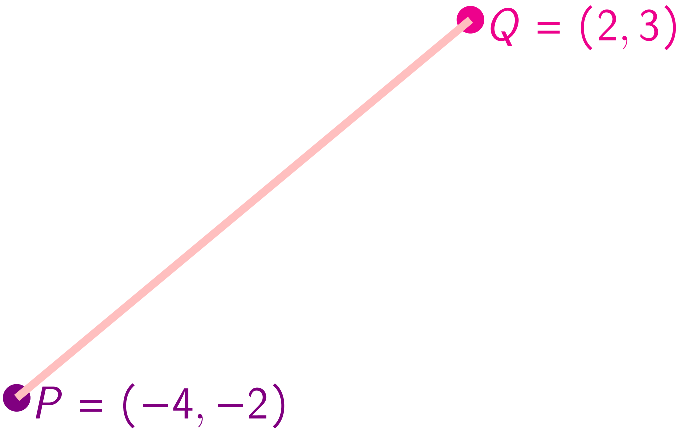
provided $x_1 \neq x_0$.

Think about what happens when $x_1 = x_0$.

What is the Slope?



Example of Computing the Slope of a Line

The **slope** (or 'steepness') m of the line containing the points $P = (-4, -2)$ and $Q = (2, 3)$ is

$$m = \frac{3 - (-2)}{2 - (-4)}$$

Example of Computing the Slope of a Line

The **slope** (or 'steepness') m of the line containing the points $P = (-4, -2)$ and $Q = (2, 3)$ is

$$m = \frac{3 - (-2)}{2 - (-4)} = \frac{3 + 2}{2 + 4}$$

Example of Computing the Slope of a Line

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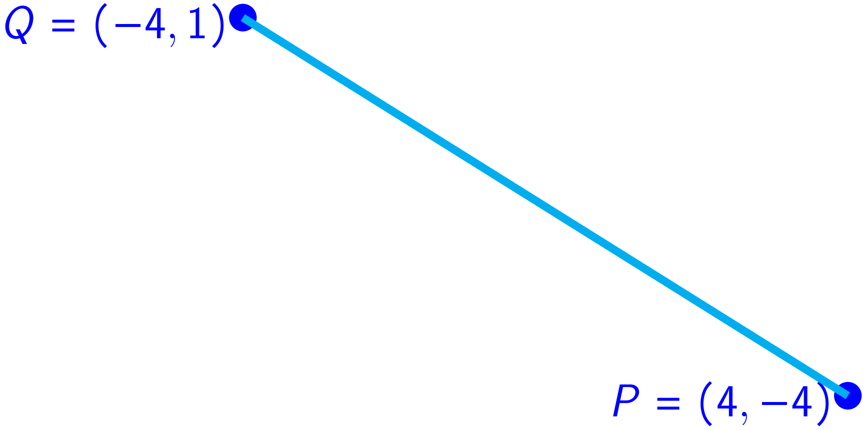
$$\boxed{m = \frac{3 - (-2)}{2 - (-4)} = \frac{3 + 2}{2 + 4} = \frac{5}{6}}$$

What is the Slope?



Example of Computing the Slope of a Line

The **slope** (or 'steepness') m of the line containing the points $P = (4, -4)$ and $Q = (-4, 1)$ is

$$m = \frac{1 - (-4)}{-4 - (4)}$$

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$$\boxed{m = \frac{1 - (-4)}{-4 - (4)} = \frac{1 + 4}{-4 - 4} = -\frac{5}{8}}$$

What is the Slope?

Example of Computing the Slope of a Line

The **slope** (or 'steepness') m of the line containing the points $P = (-2, 2)$ and $Q = (2, 2)$ is

$$m = \frac{2 - (2)}{2 - (-2)}$$

Example of Computing the Slope of a Line

The **slope** (or 'steepness') m of the line containing the points $P = (-2, 2)$ and $Q = (2, 2)$ is

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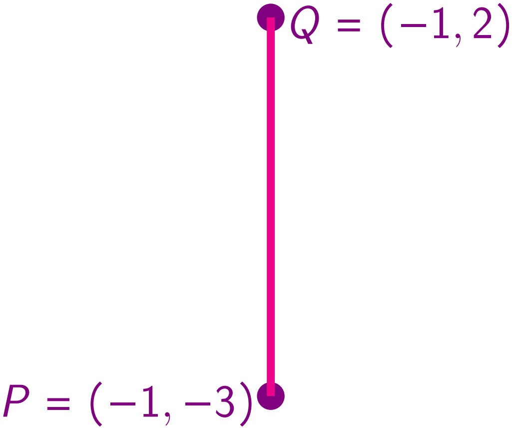
Example of Computing the Slope of a Line

The **slope** (or 'steepness') m of the line containing the points $P = (-2, 2)$ and $Q = (2, 2)$ is

$$\boxed{m = \frac{2 - (2)}{2 - (-2)} = \frac{2 - 2}{2 + 2} = \frac{0}{4} = 0}$$

What is the Slope?



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The **slope** (or 'steepness') m of the line containing the points $P = (-1, -3)$ and $Q = (-1, 2)$ is

$$\boxed{m = \frac{2-(-3)}{-1-(-1)} = \frac{2+3}{-1+1} = \frac{5}{0} = \infty \text{ or undefined}}$$

Slope is the Rate of Change Δ

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$$\Delta y = y_1 - y_0 \quad \text{(change in } y\text{)}$$

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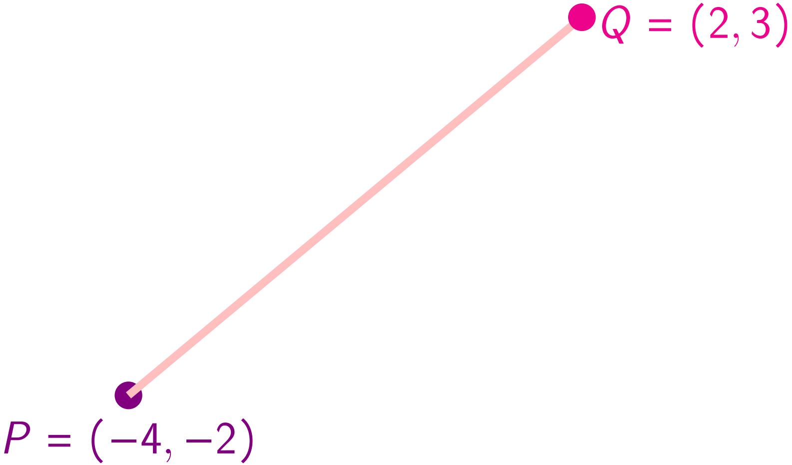
$$m = \frac{\Delta y}{\Delta x}$$

Rate of Change of y with respect to x.



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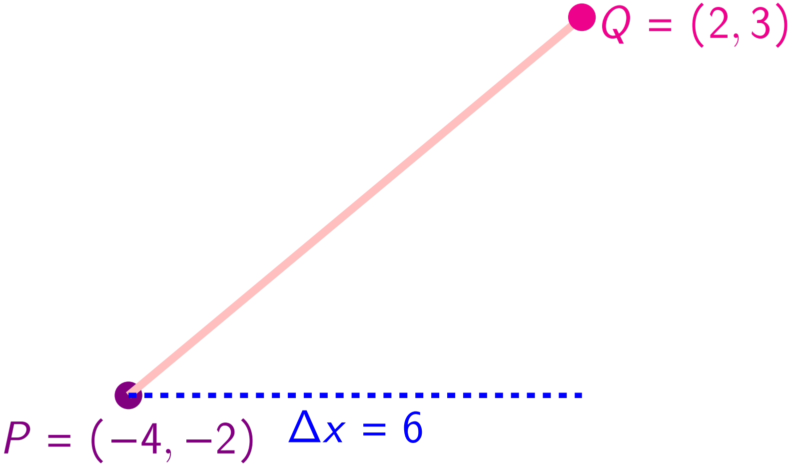

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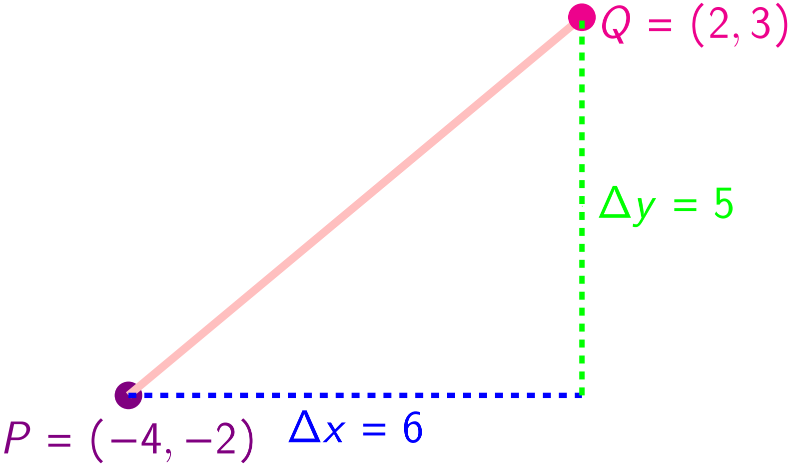

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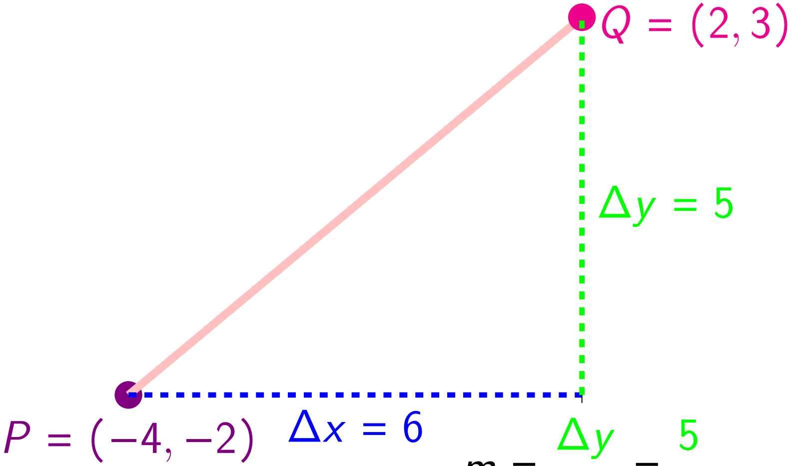


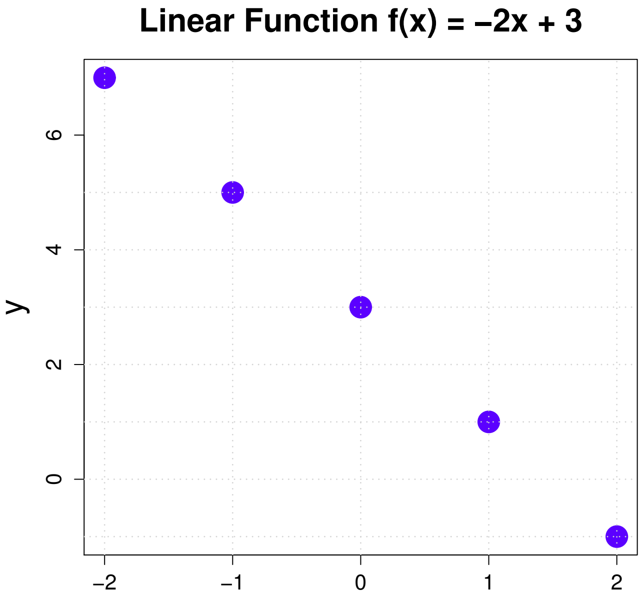
$$m = \frac{\Delta y}{\Delta x} = \frac{5}{6}$$

A linear function is a function of the form

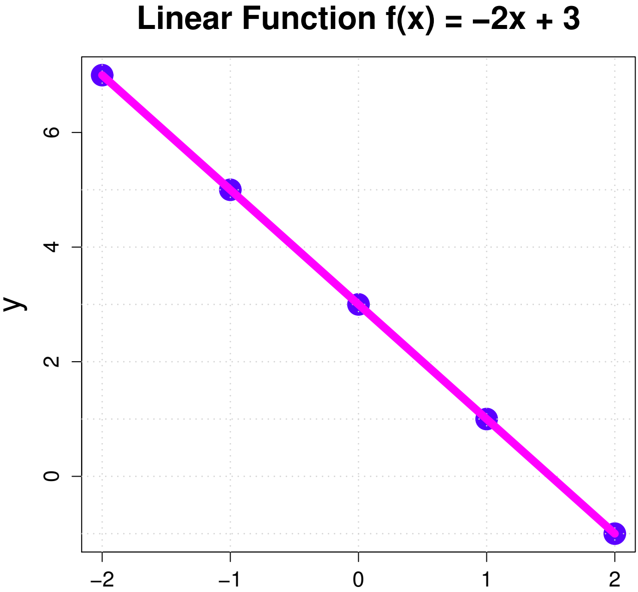
$$f(x) = mx + b$$

where m and b are real numbers. The domain of a linear function is $(-\infty, \infty)$.

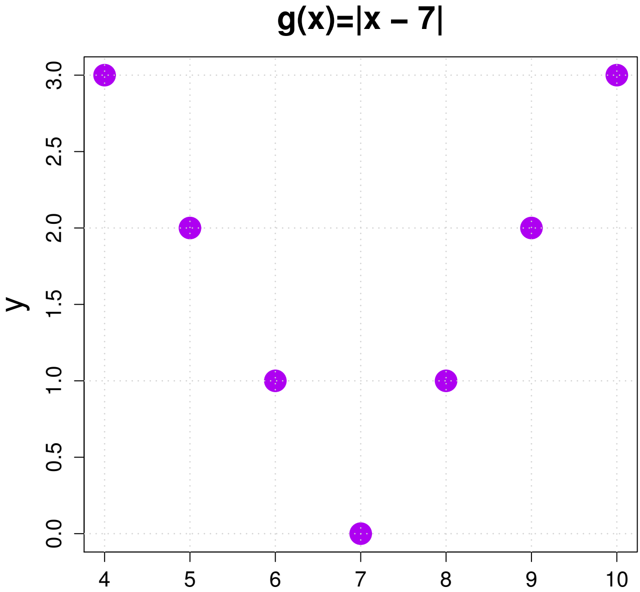
Plotted Points for Function $f(x) = -2x + 3$



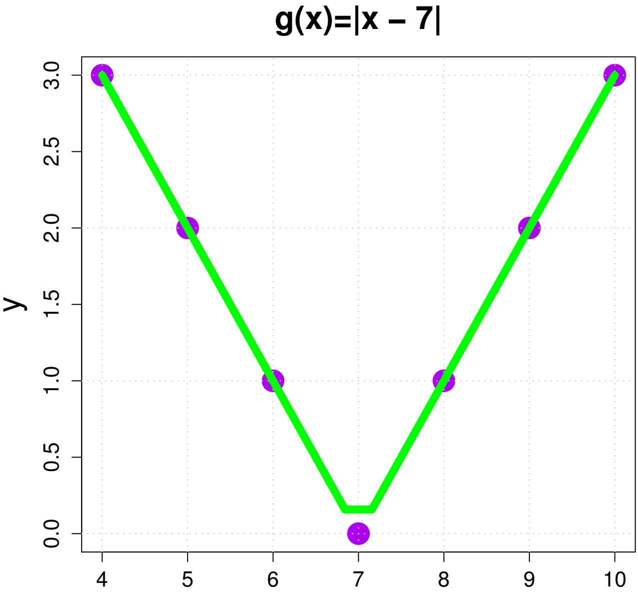
Graph of Function $f(x) = -2x + 3$



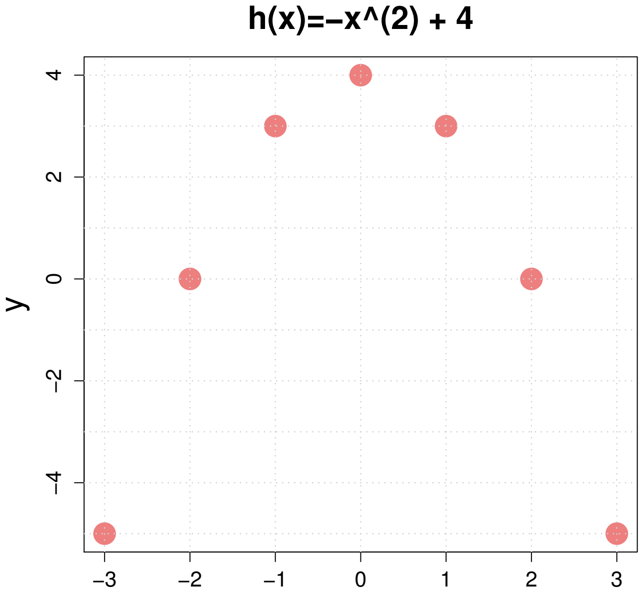
Plotted Points for Function $g(x) = |x - 7|$



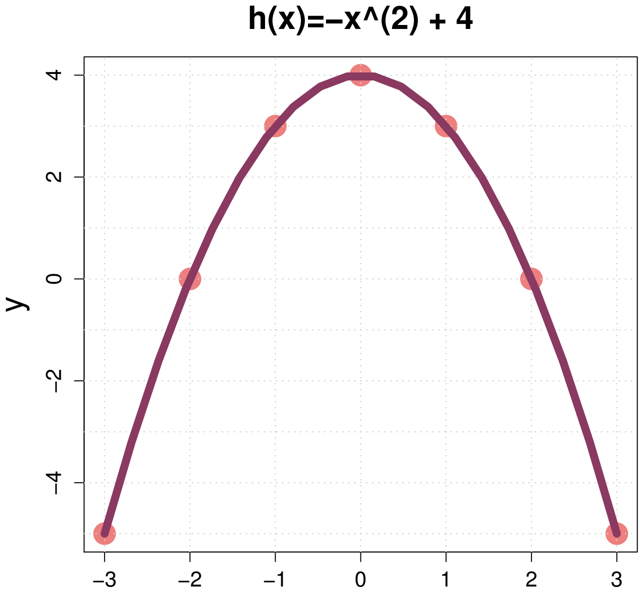
Graph of Function $g(x) = |x - 7|$ (nonlinear function)



Plotted Points for Function $h(x) = 4 + -x^2$



Graph of Function $h(x) = 4 + -x^2$ (nonlinear function)



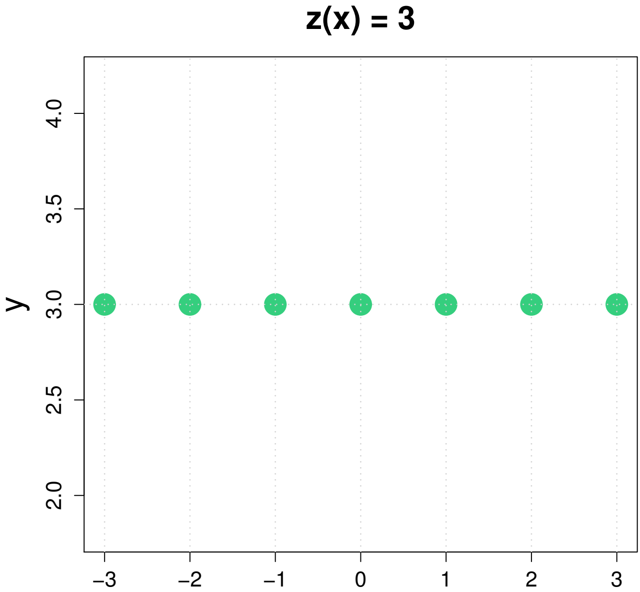
A **constant function** is a function of the

form

$$f(x) = b$$

where b is a real numbers. The domain of a linear function is $(-\infty, \infty)$.

Plotted Points for Function $z(x) = 3$



Graph of Constant Function $z(x) = 3$