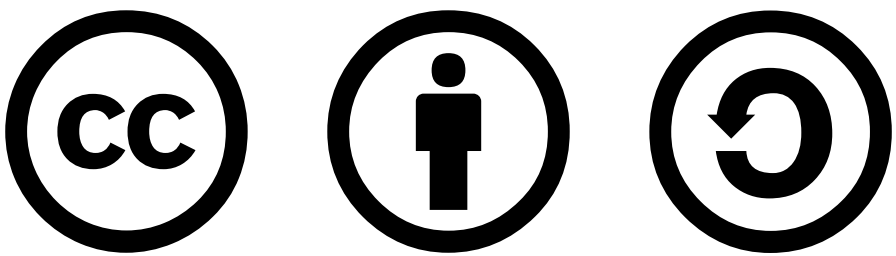


Algebra Lecture 4

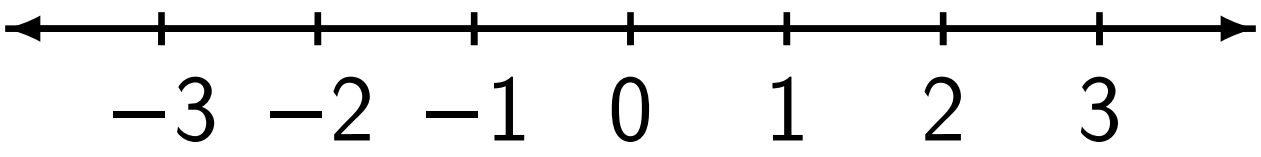
Crista Moreno

December 11, 2016

Topics for Today

1. Relations
2. Domains
3. Ranges
4. Functions

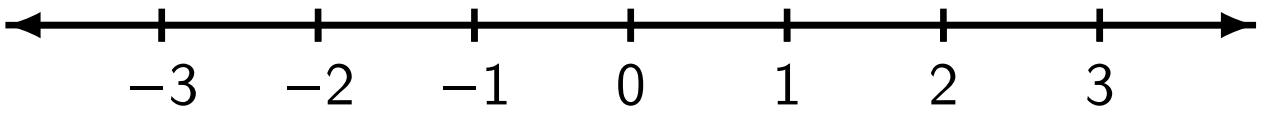
Real Number Line $\mathbb{R}$



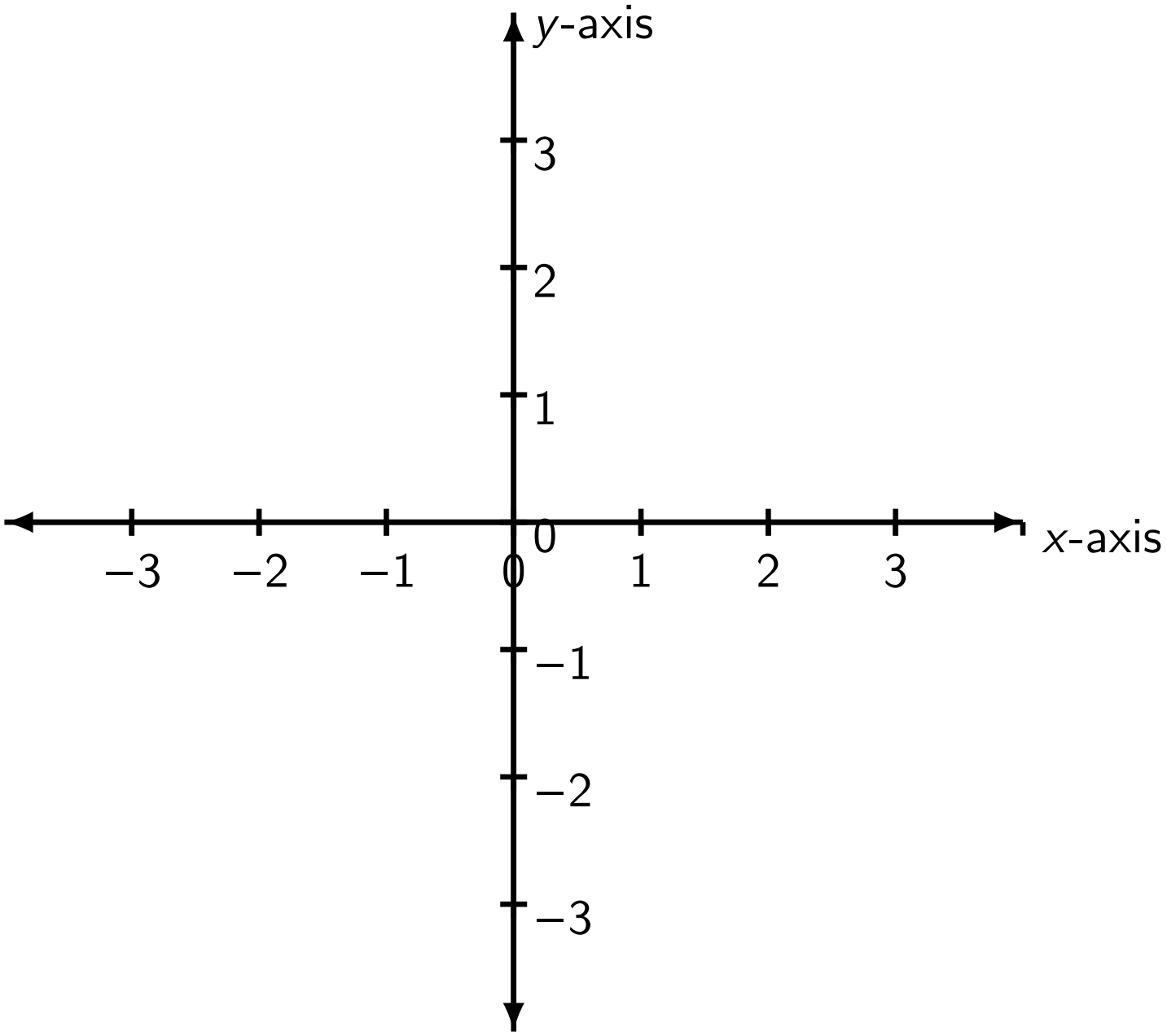
Cartesian Coordinate Plane

Imagine two copies of the real number line $\mathbb{R}$ crossing at a right angle at the origin 0. The horizontal line is called the x-axis and the vertical line is called the y-axis.

Cartesian Plane $\mathbb{R}^2$



Cartesian Plane $\mathbb{R}^2$



Points in the Cartesian Coordinate Plane

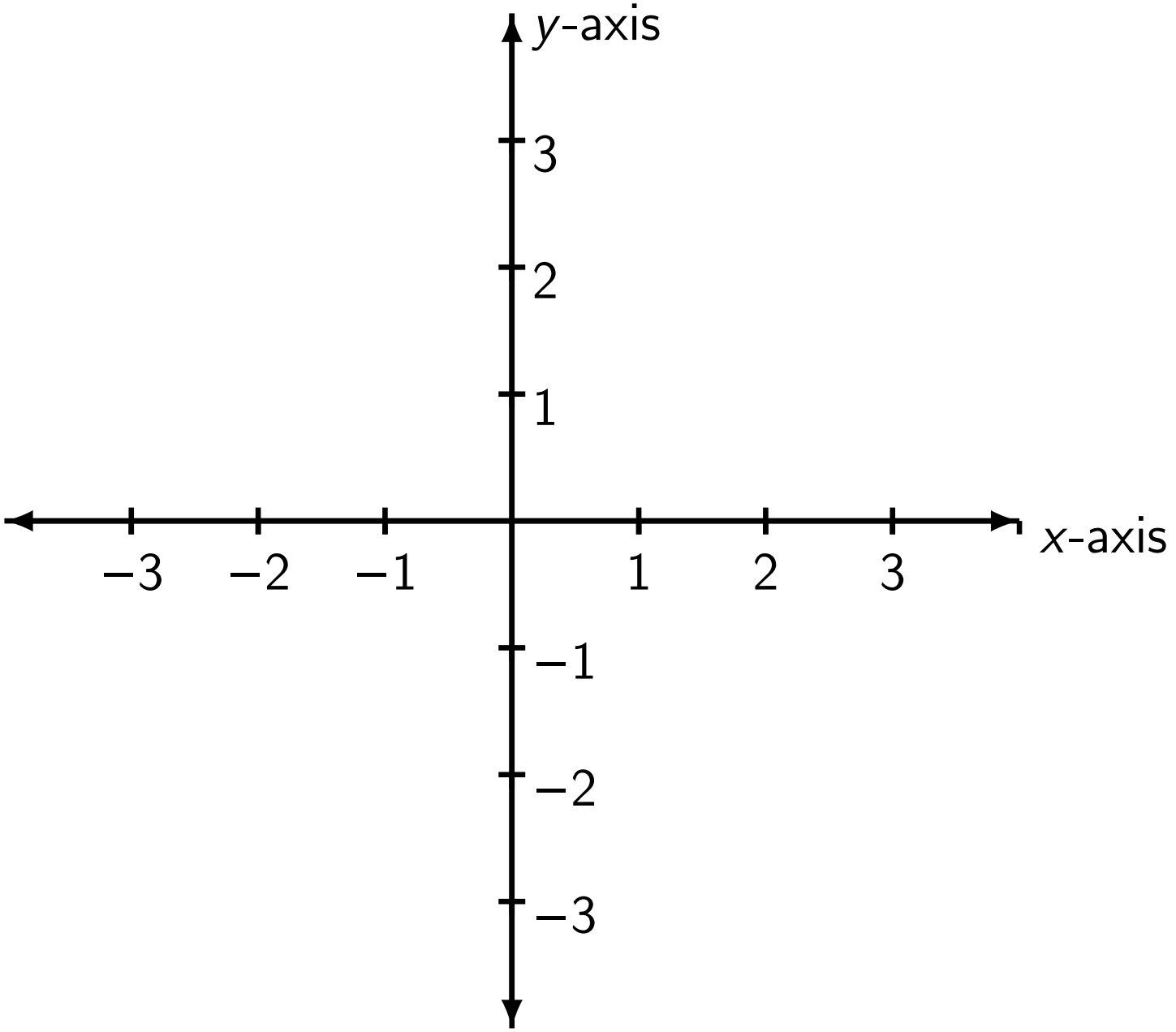
We describe the **point** P using the **ordered pair** $(2, -3)$. The first entry in the ordered pair is called the **abscissa** or x-**coordinate** and the second is called the **ordinate** or y-**coordinate**. Together they comprise the **Cartesian coordinates** of the point P.

Points in the Cartesian Coordinate Plane

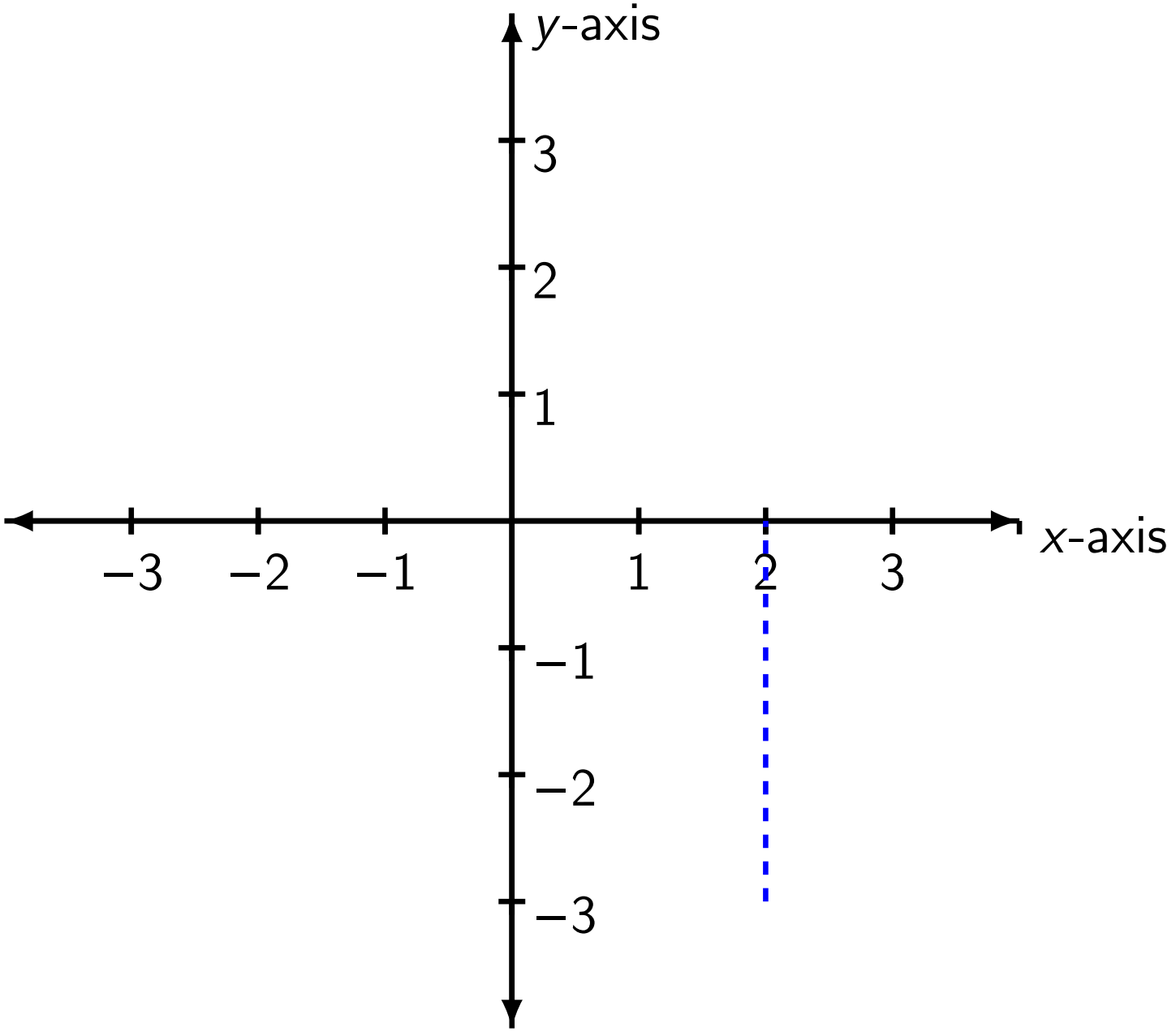
We describe the **point** P using the **ordered pair** $(2, -3)$. The first entry in the ordered pair is called the **abscissa** or x-**coordinate** and the second is called the **ordinate** or y-**coordinate**. Together they comprise the **Cartesian coordinates** of the point P.

How would you plot the point $P = (2, -3)$?

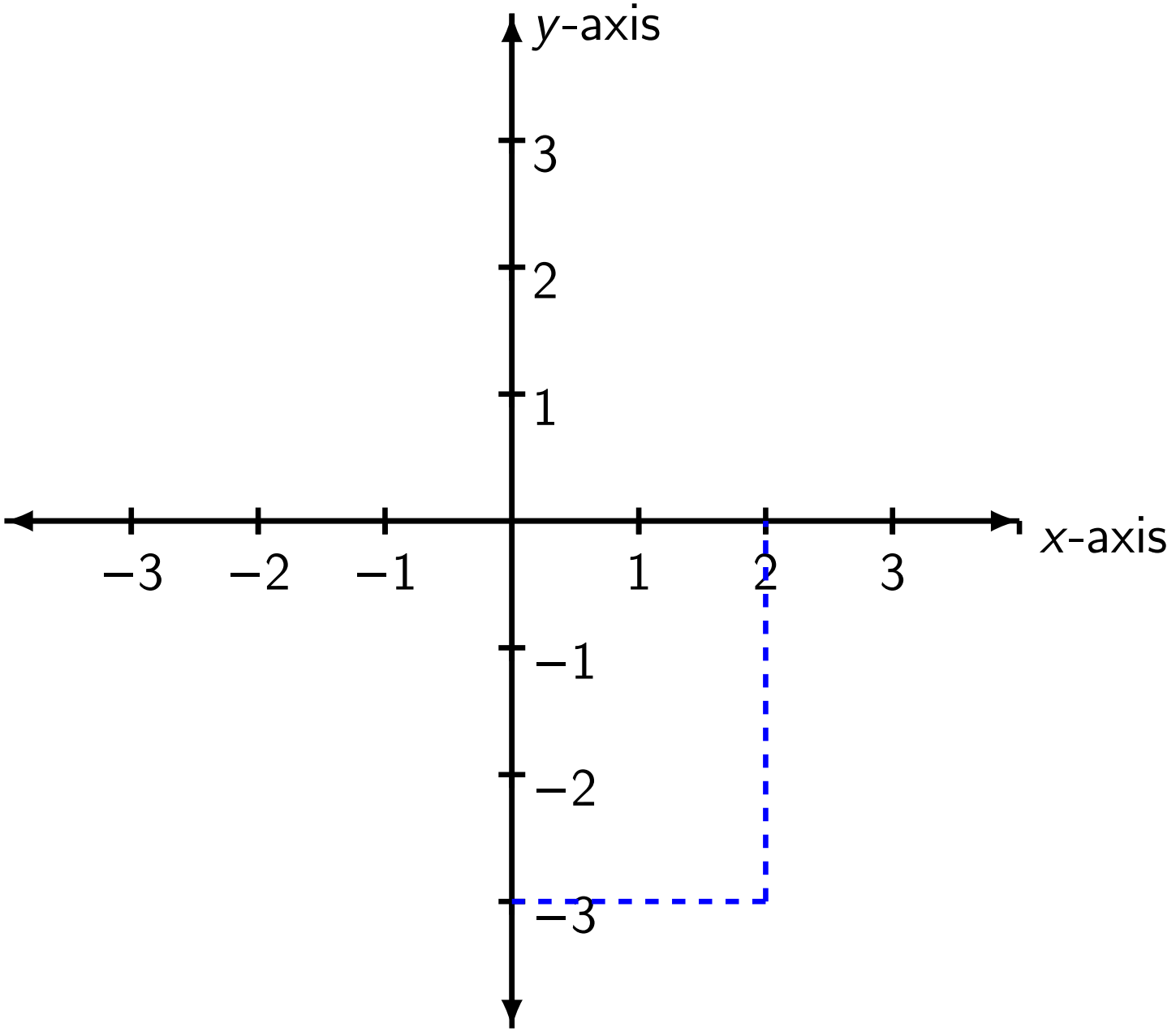
Plot Points $(2,-3)$, $(-1,2)$, $(-3,-2)$ and $(1,1)$ in $\mathbb{R}^2$



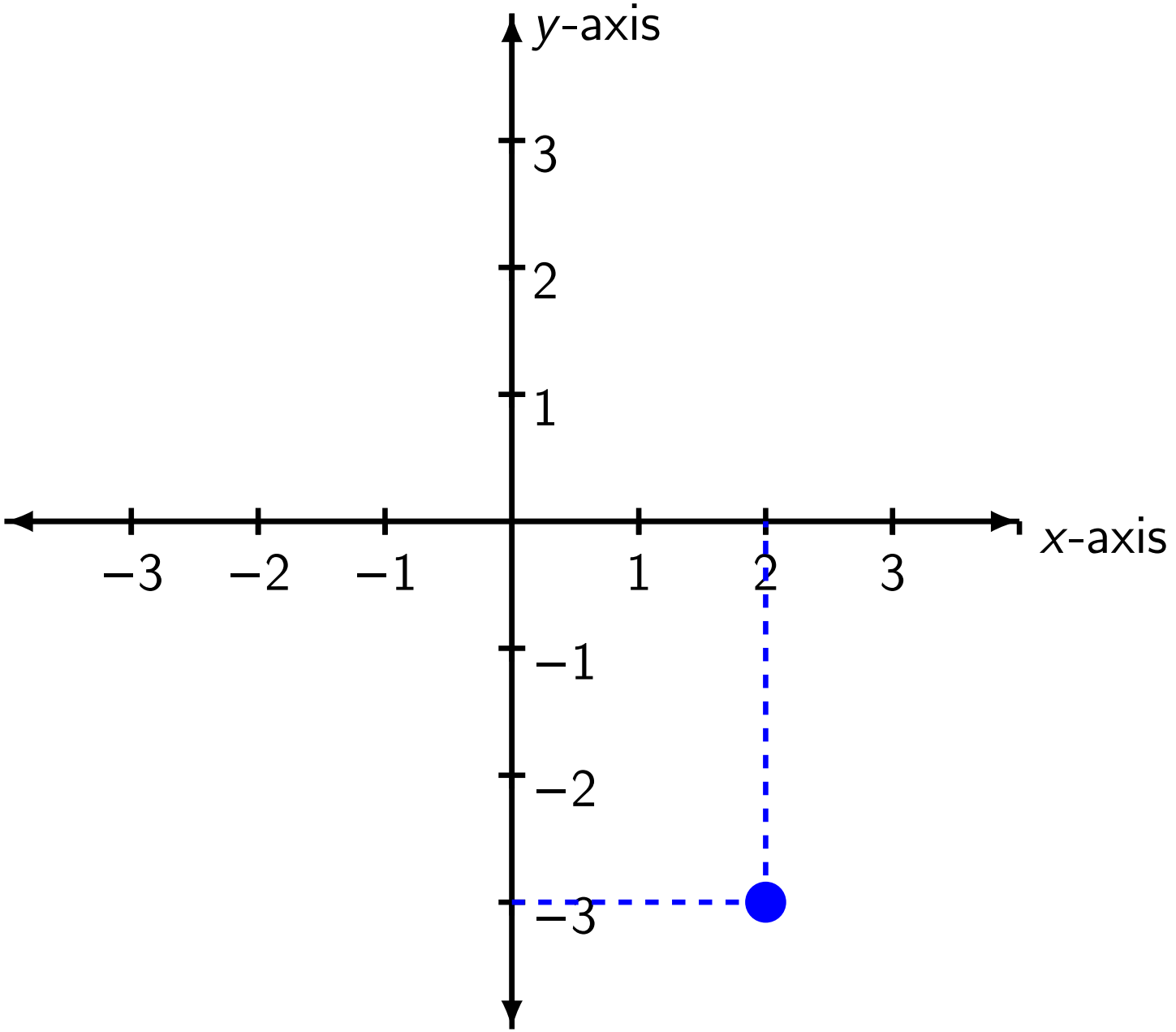
Plot Points $(2,-3)$, $(-1,2)$, $(-3,-2)$ and $(1,1)$ in $\mathbb{R}^2$



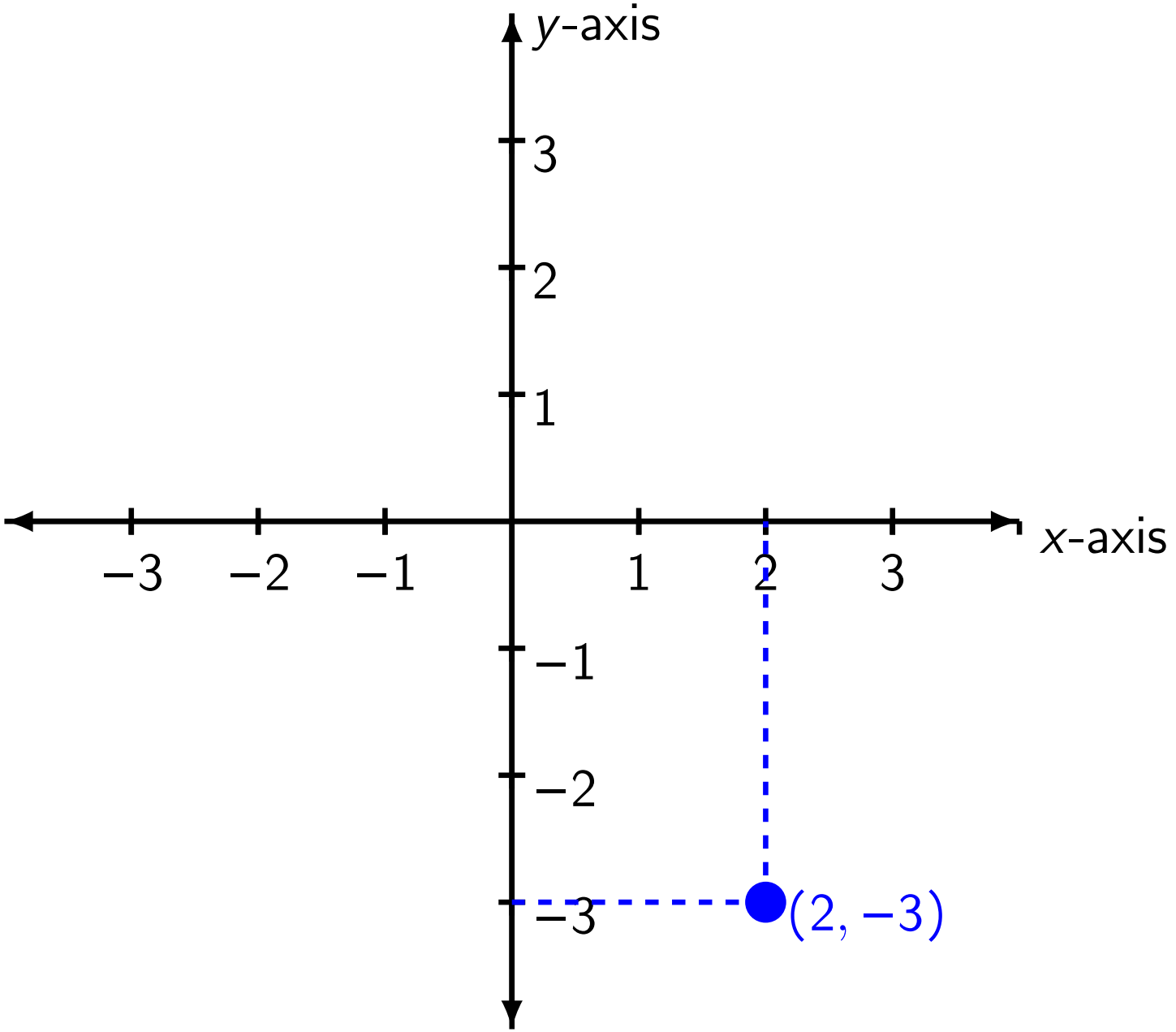
Plot Points $(2,-3)$, $(-1,2)$, $(-3,-2)$ and $(1,1)$ in $\mathbb{R}^2$



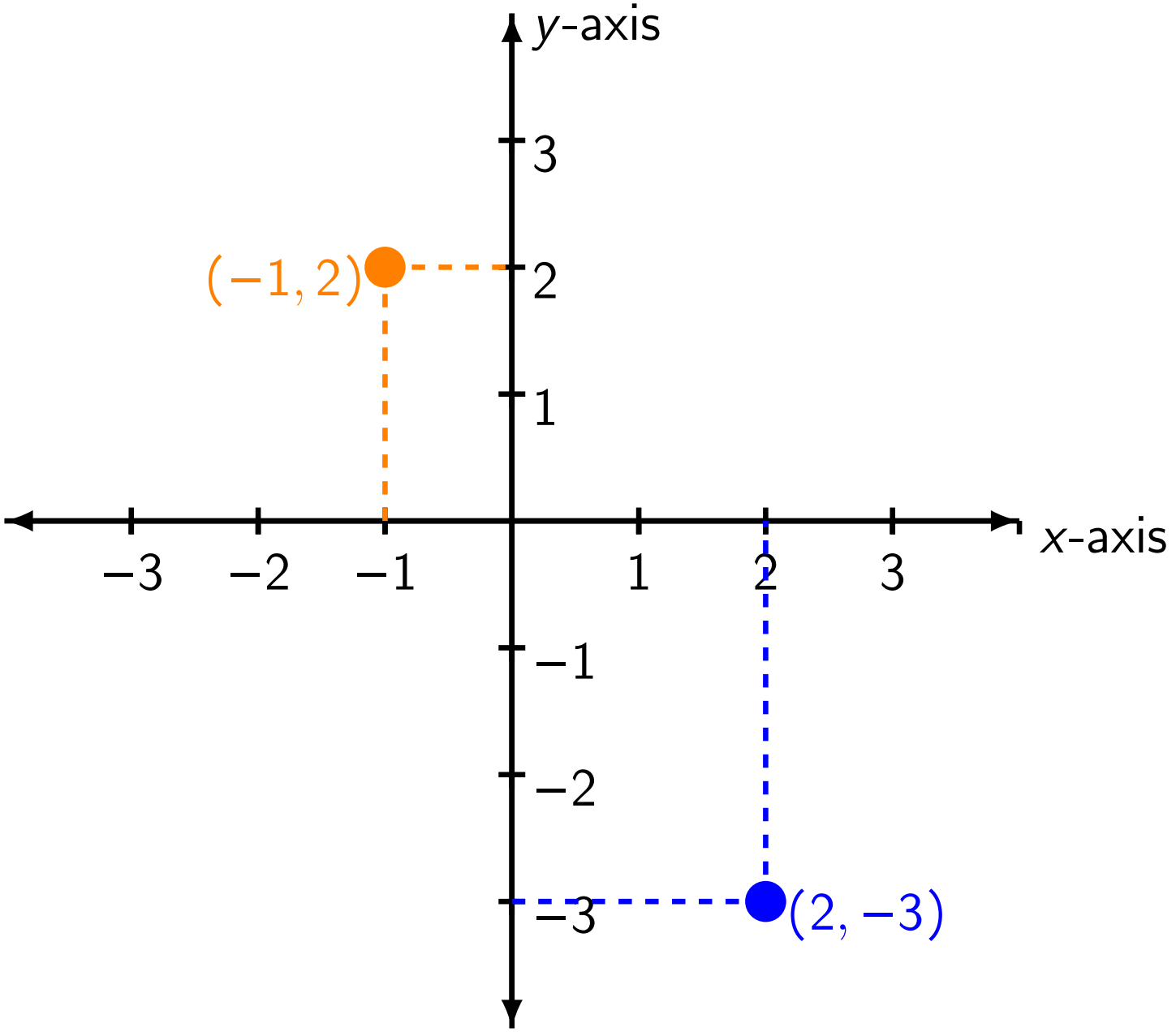
Plot Points $(2,-3)$, $(-1,2)$, $(-3,-2)$ and $(1,1)$ in $\mathbb{R}^2$



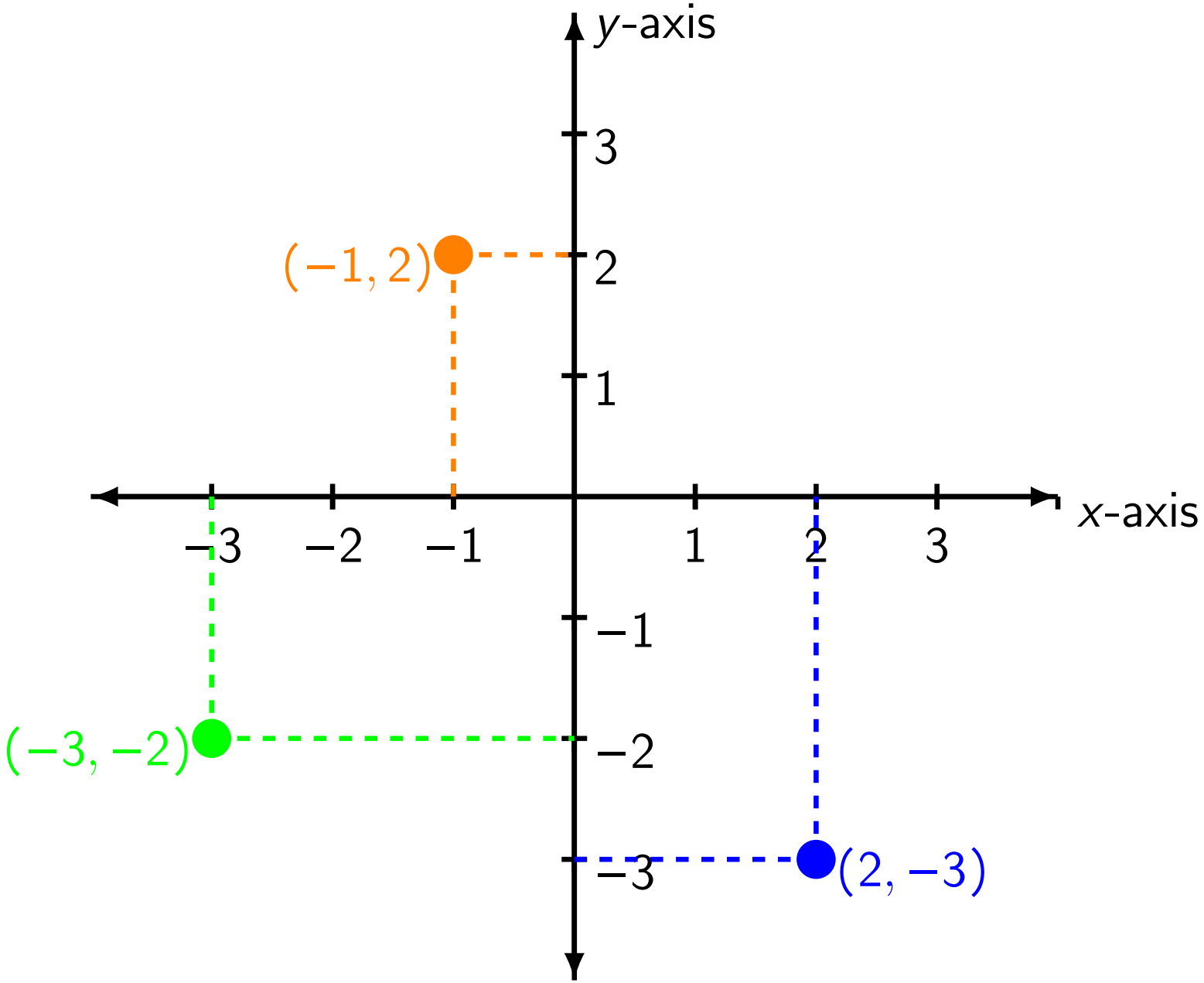
Plot Points $(2,-3)$, $(-1,2)$, $(-3,-2)$ and $(1,1)$ in $\mathbb{R}^2$



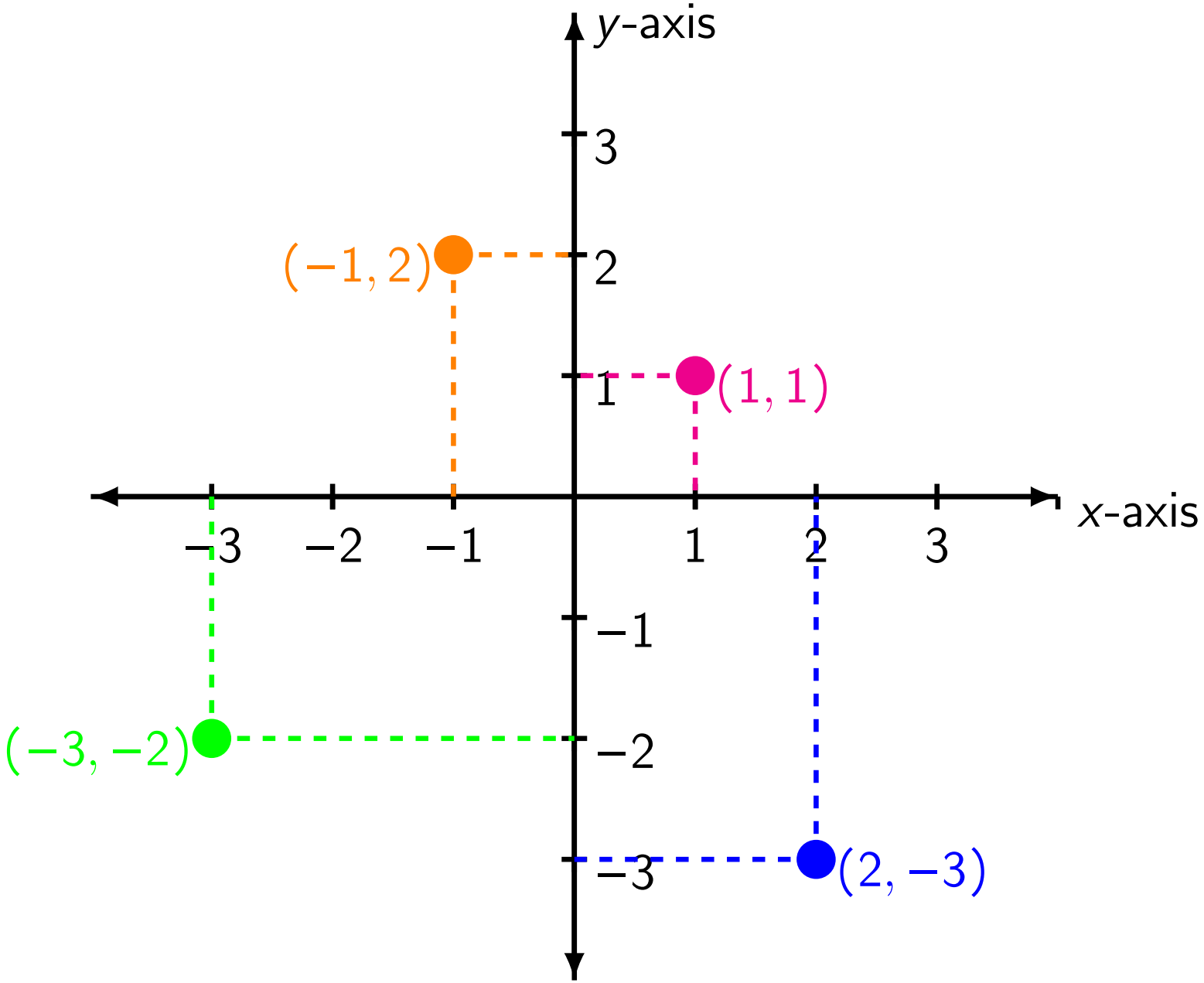
Plot Points $(2,-3)$, $(-1,2)$, $(-3,-2)$ and $(1,1)$ in $\mathbb{R}^2$



Plot Points $(2,-3)$, $(-1,2)$, $(-3,-2)$ and $(1,1)$ in $\mathbb{R}^2$



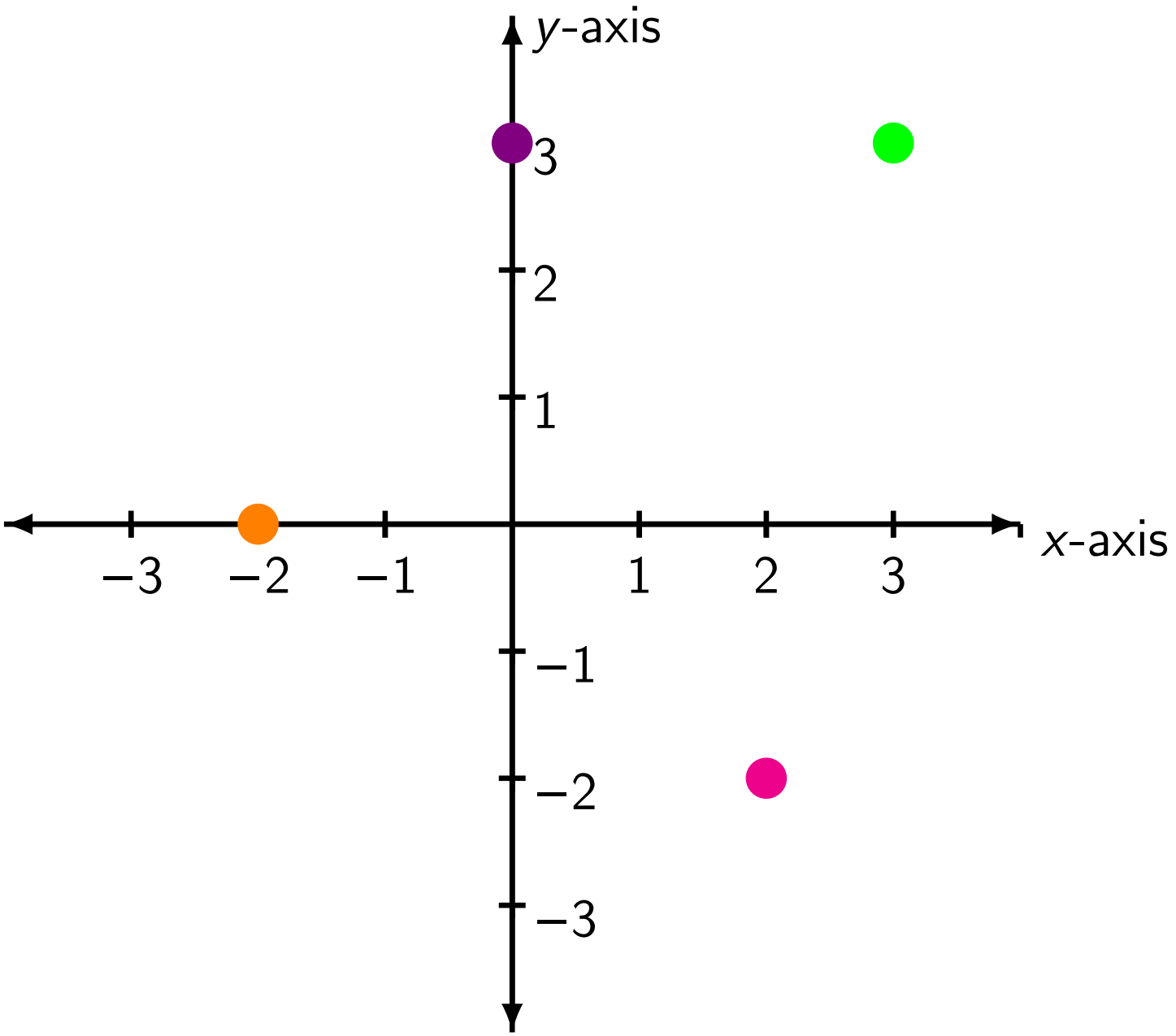
Plot Points $(2,-3)$, $(-1,2)$, $(-3,-2)$ and $(1,1)$ in $\mathbb{R}^2$



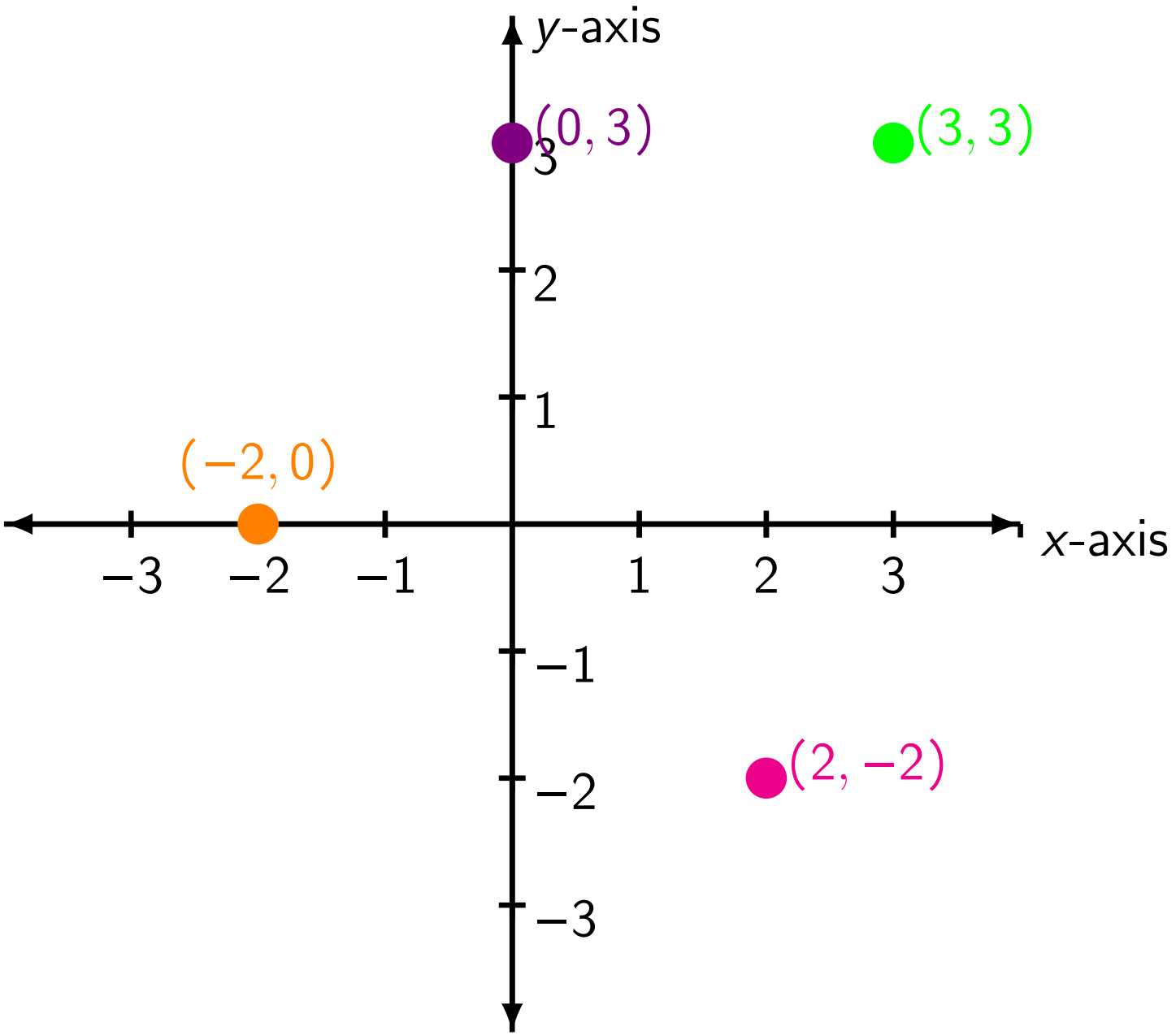
Plotting Points in the Cartesian Plane $\mathbb{R}^2$

What points are plotted in $\mathbb{R}^2$?

Plot Points in $\mathbb{R}^2$



Plotted Points in $\mathbb{R}^2$ $(3,3), (0,3), (-2,0), (2,-2)$



Important Facts

Facts about the Cartesian Coordinate Plane

Important Facts

Facts about the Cartesian Coordinate Plane

- (a, b) and (c, d) represent the same point in the plane if and only if $a = c$ and $b = d$.

Important Facts

Facts about the Cartesian Coordinate Plane

- (a, b) and (c, d) represent the same point in the plane if and only if $a = c$ and $b = d$.
- (x, y) lies on the x-axis if and only if $y = 0$.

Important Facts

Facts about the Cartesian Coordinate Plane

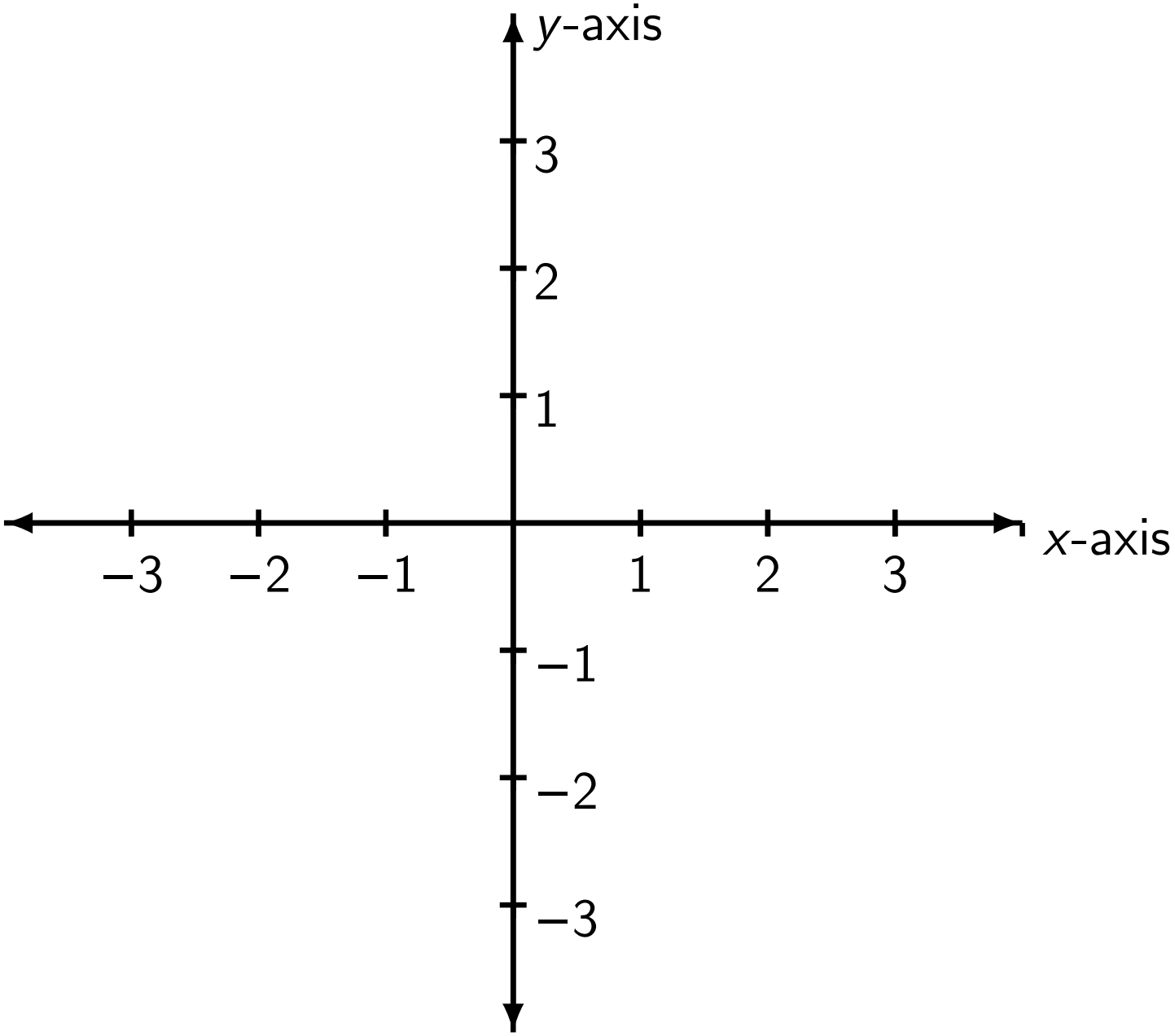
- (a, b) and (c, d) represent the same point in the plane if and only if $a = c$ and $b = d$.
- (x, y) lies on the x-axis if and only if $y = 0$.
- (x, y) lies on the y-axis if and only if $x = 0$.

Important Facts

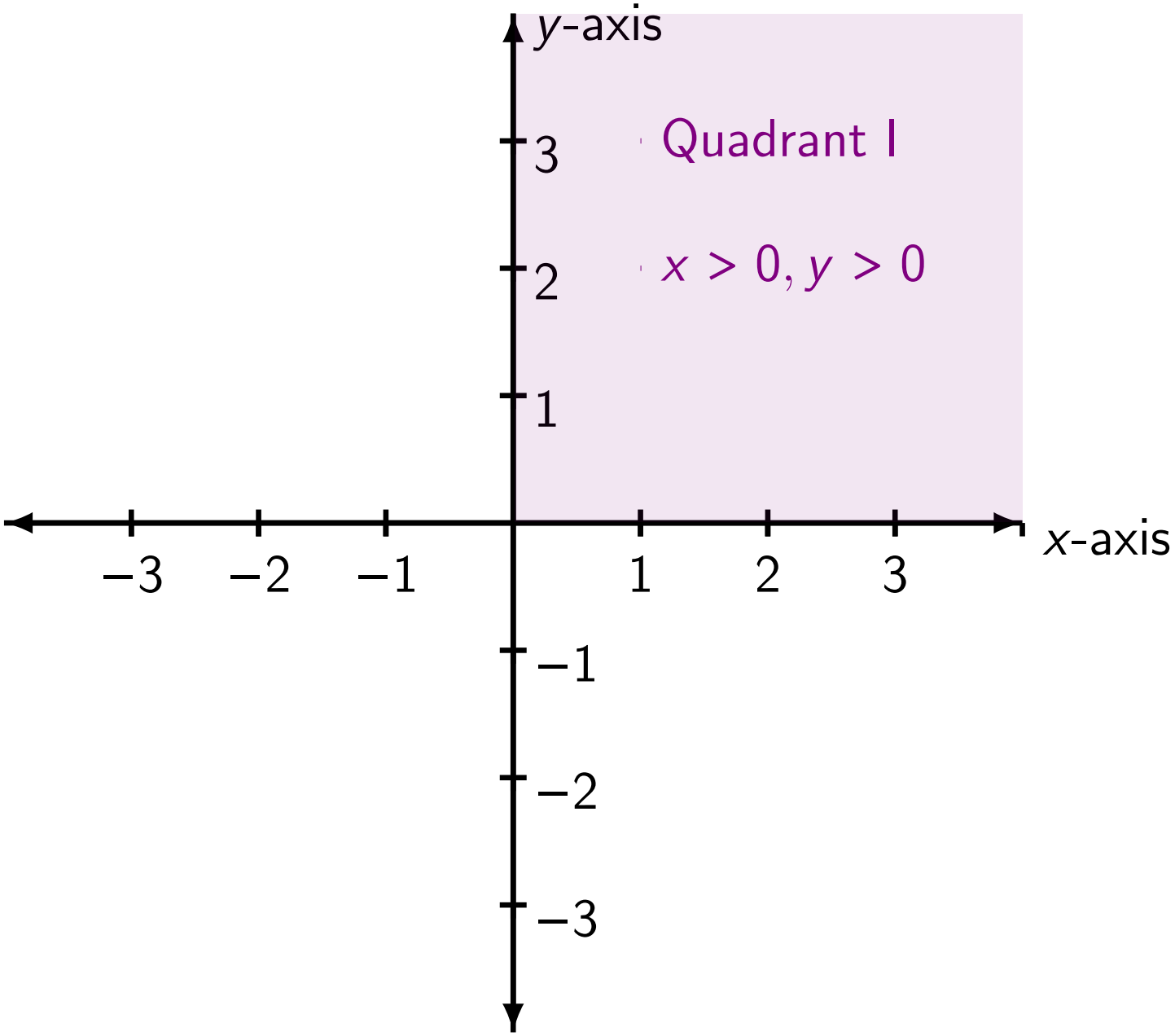
Facts about the Cartesian Coordinate Plane

- (a, b) and (c, d) represent the same point in the plane if and only if $a = c$ and $b = d$.
- (x, y) lies on the x-axis if and only if $y = 0$.
- (x, y) lies on the y-axis if and only if $x = 0$.
- The **origin** is the point $(0, 0)$. It is the only point common to both axes.

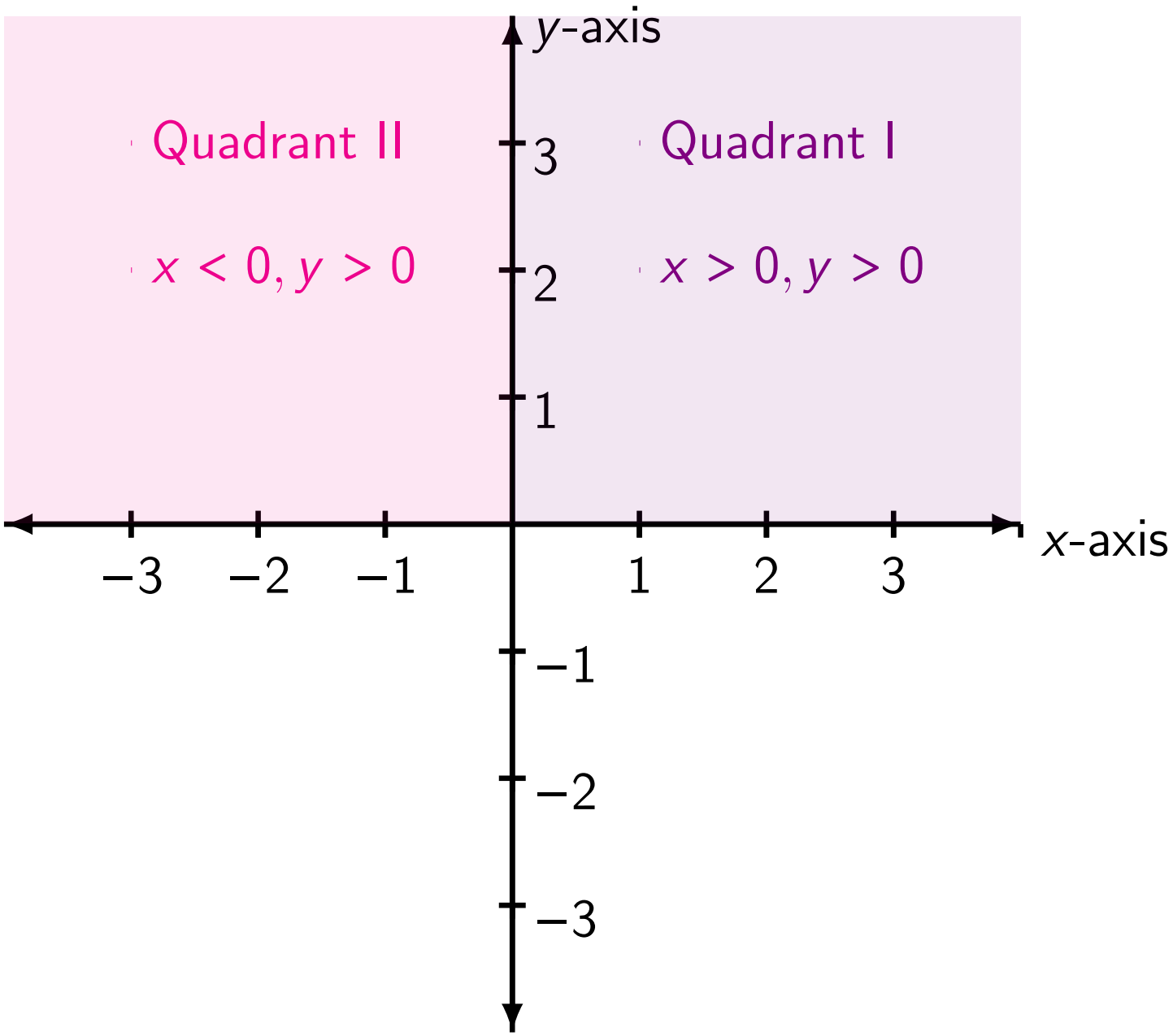
Quadrants of the Cartesian Plane



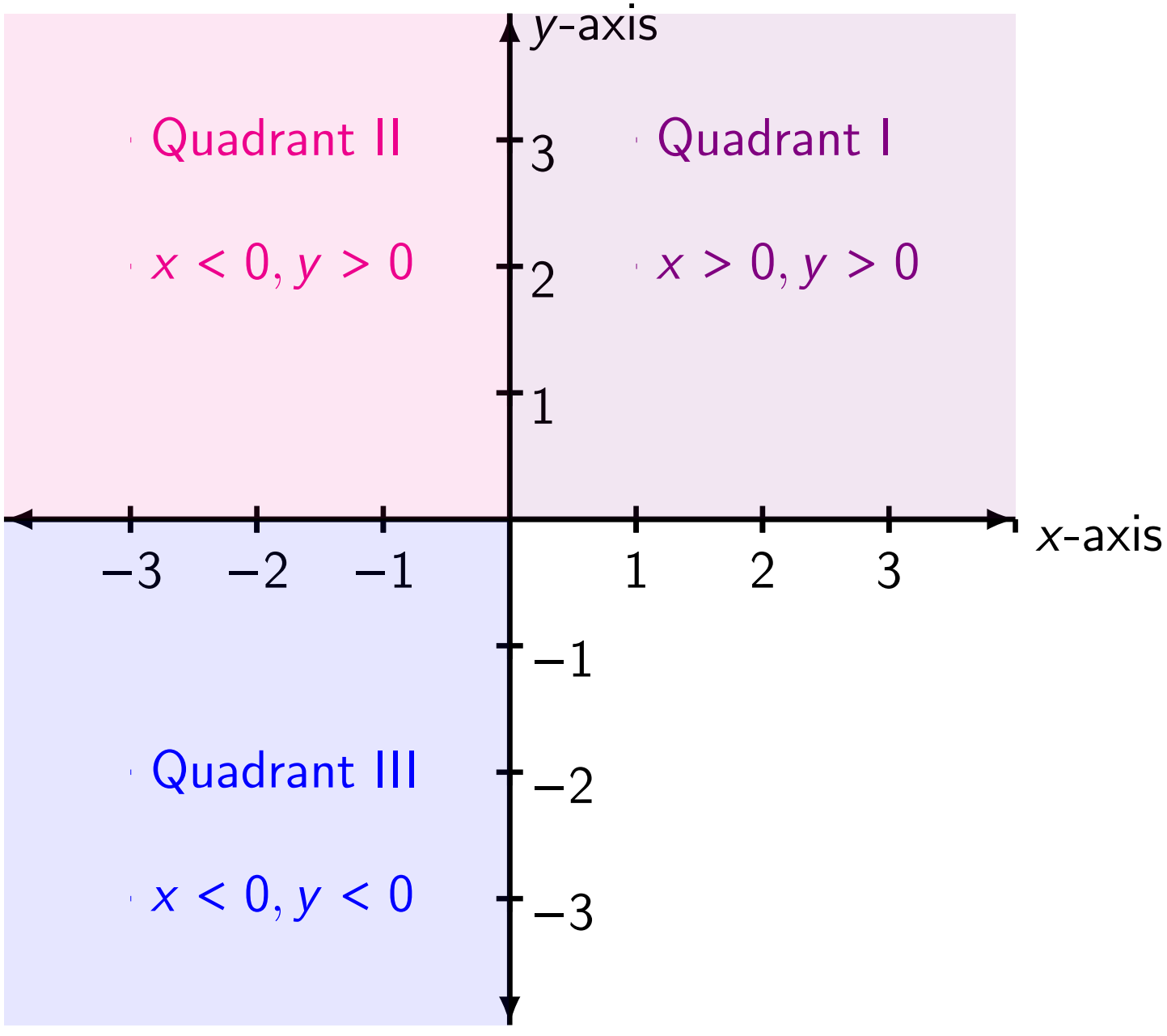
Quadrants of the Cartesian Plane



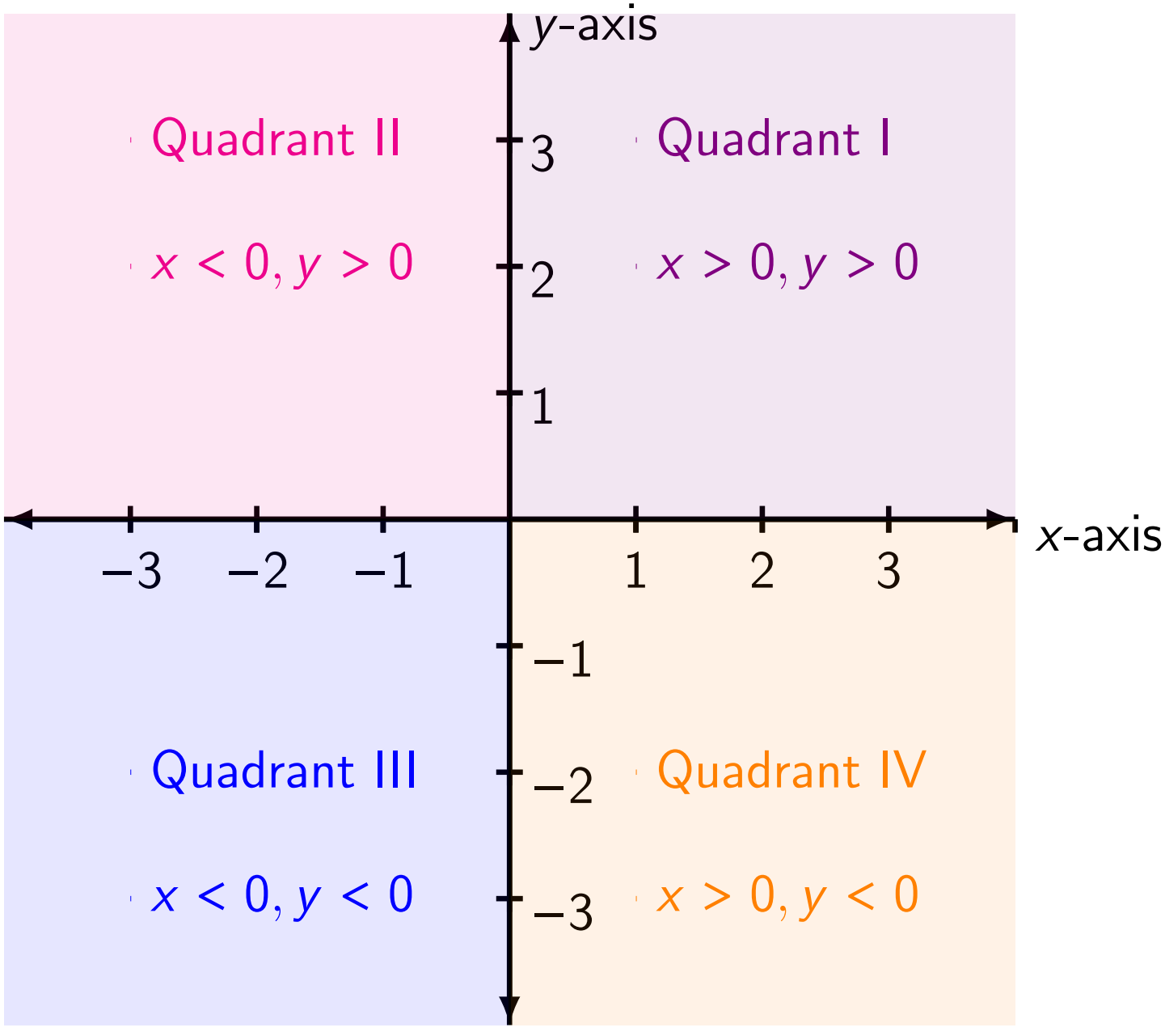
Quadrants of the Cartesian Plane



Quadrants of the Cartesian Plane



Quadrants of the Cartesian Plane



Relations

A **relation** is a set of points in the plane $\mathbb{R}^2$.

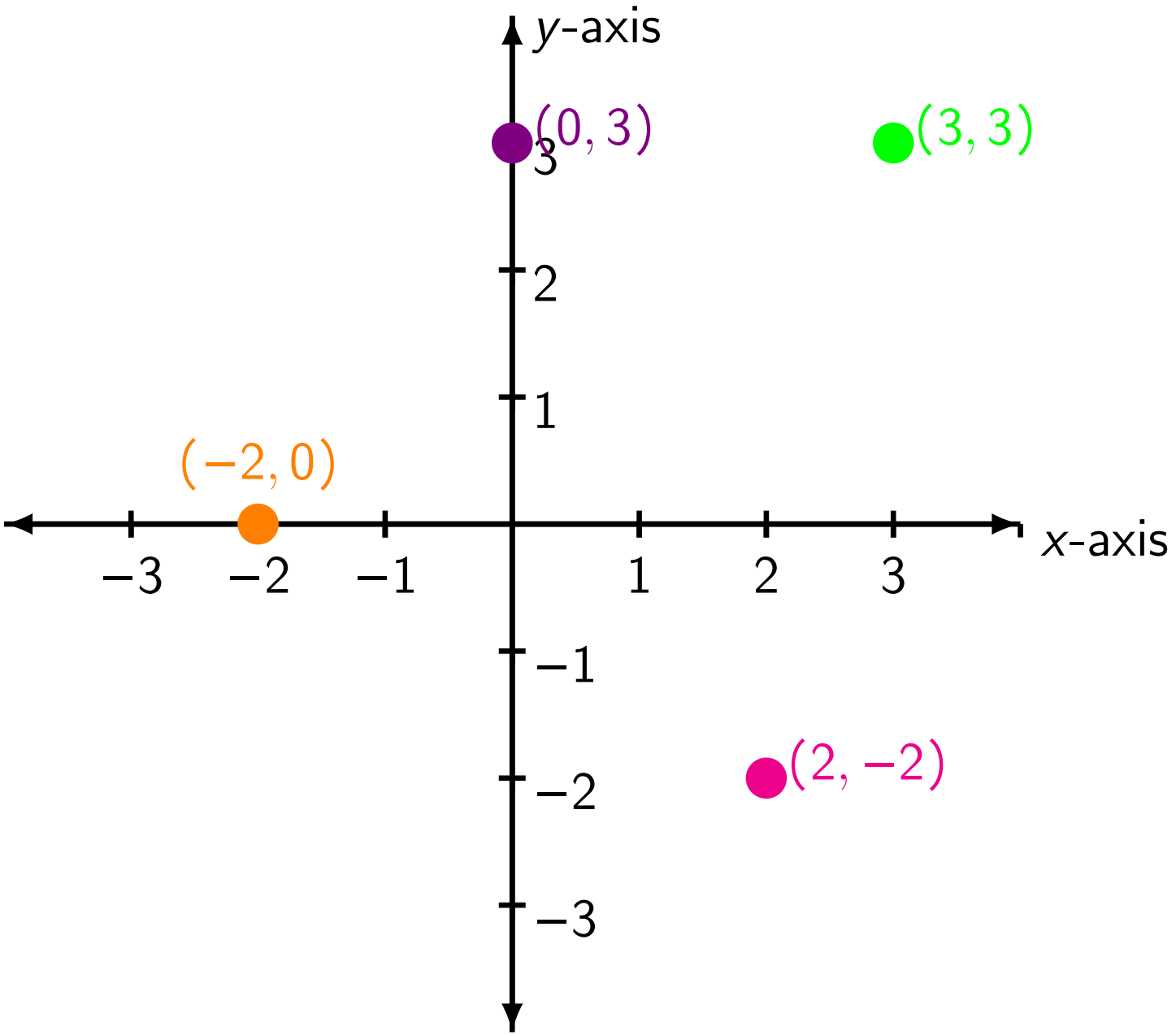
Examples of Relations

$\Gamma = \{(3,3),(0,3),(-2,0),(2,-2)\}$ is a relation.

Note: {} means 'set'

Γ gamma capital Greek letter

Relation $\Gamma = \{(3,3), (0,3), (-2,0), (2,-2)\}$



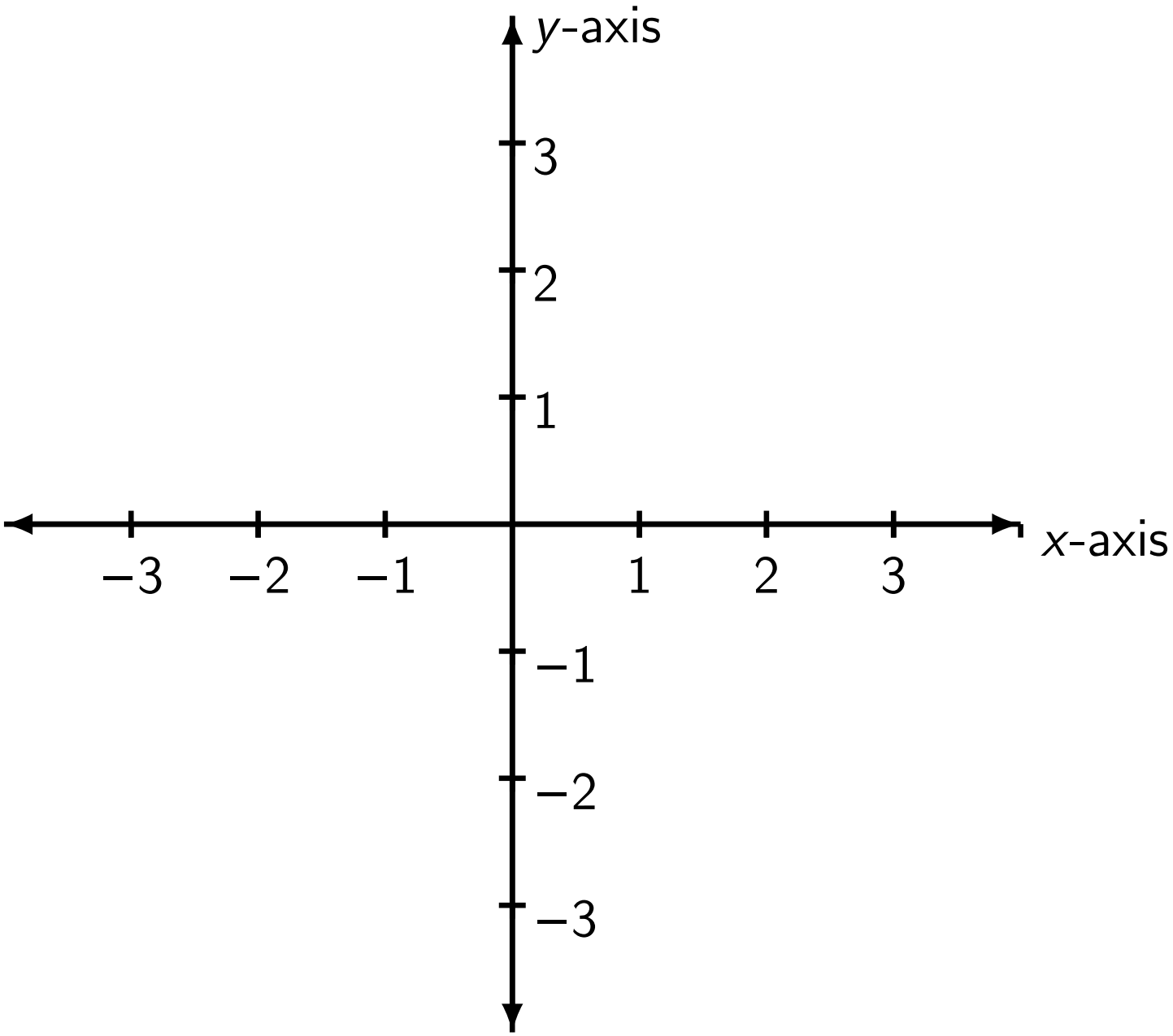
Examples of Relations

$\Omega = \{(x, 3) | -1 \leq x \leq 2\}$ is a relation.

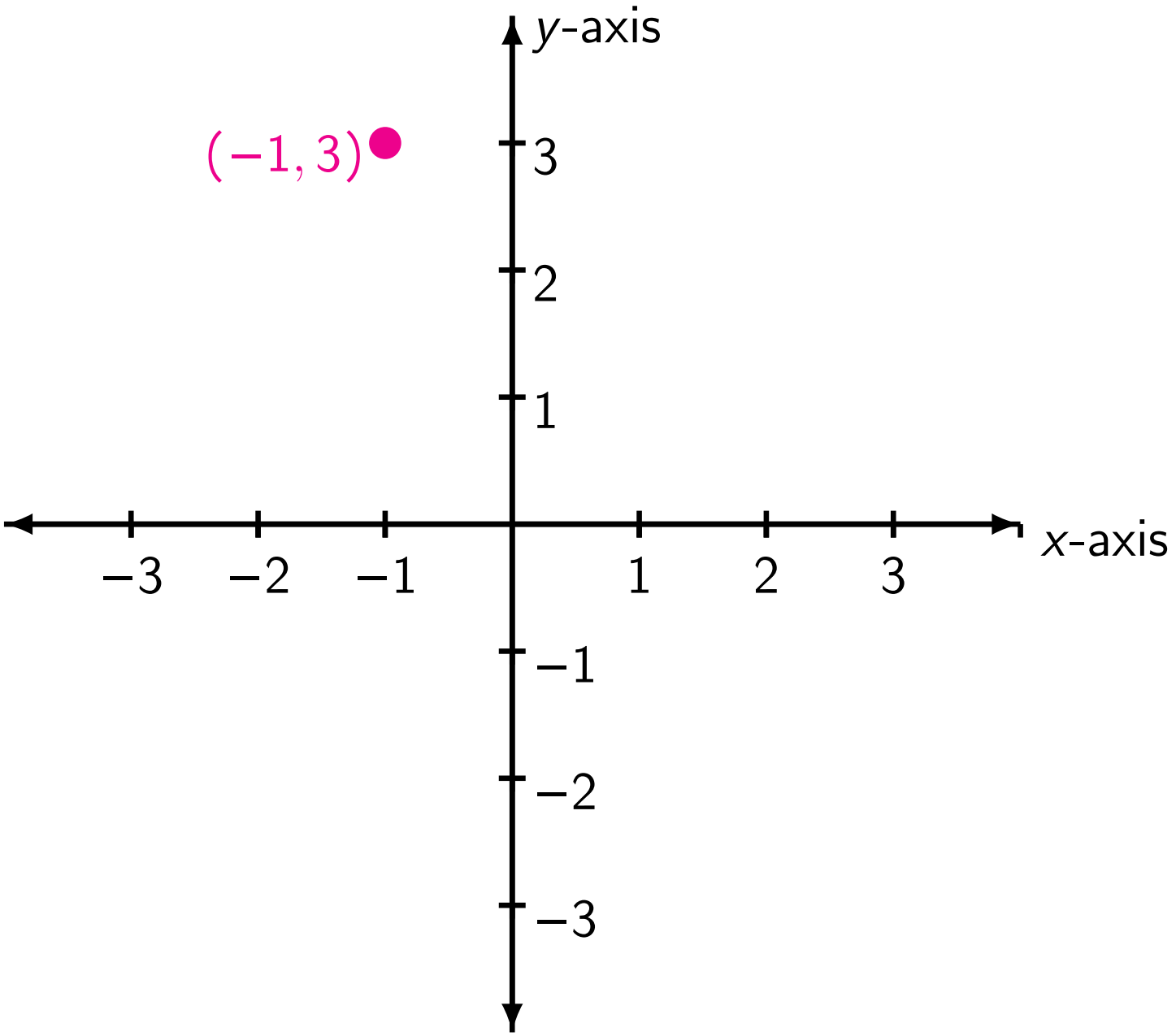
Ω-omega capital Greek letter,

| means 'such that'

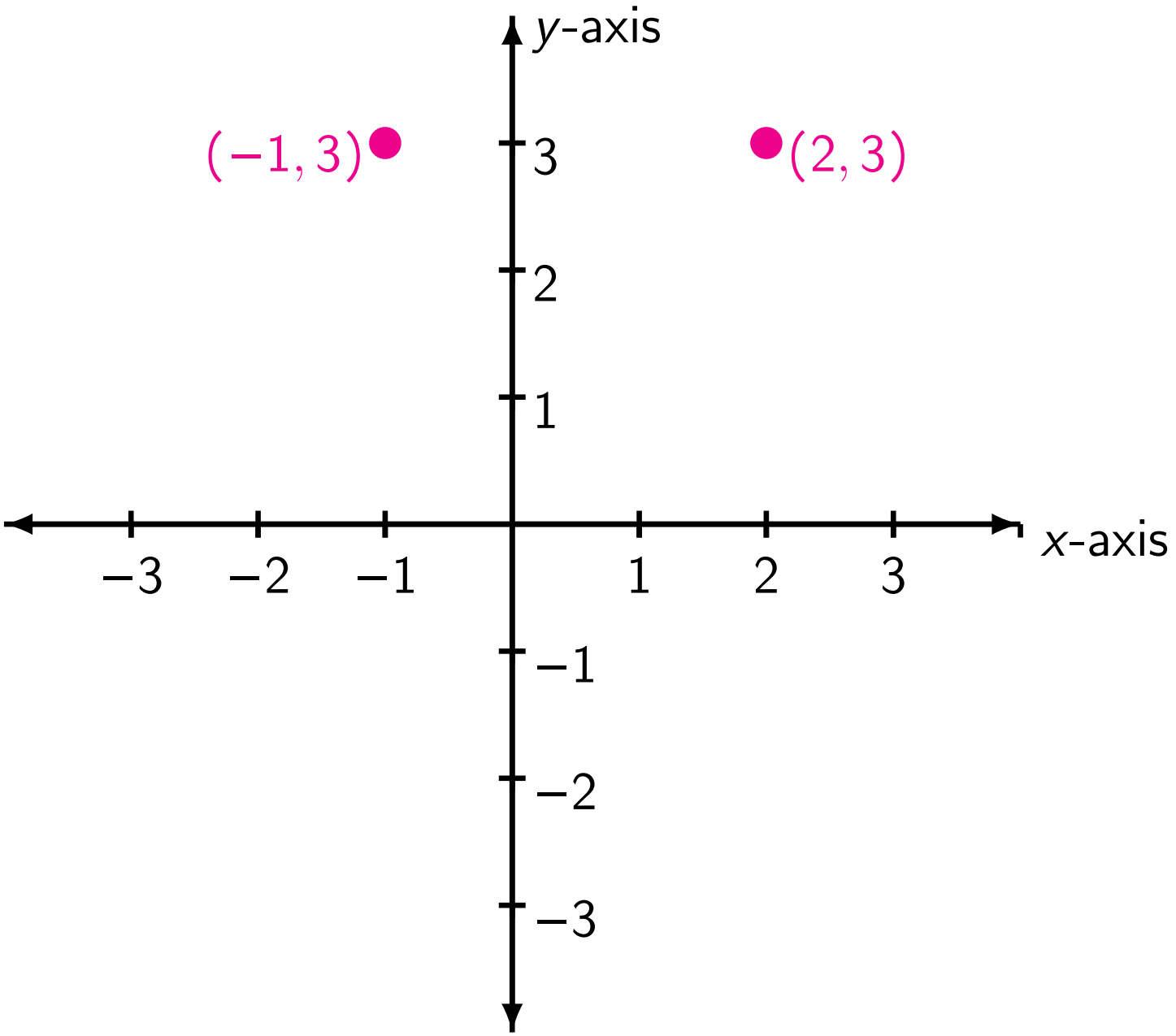
Relation $\Omega = \{(x, 3) | -1 \leq x \leq 2\}$



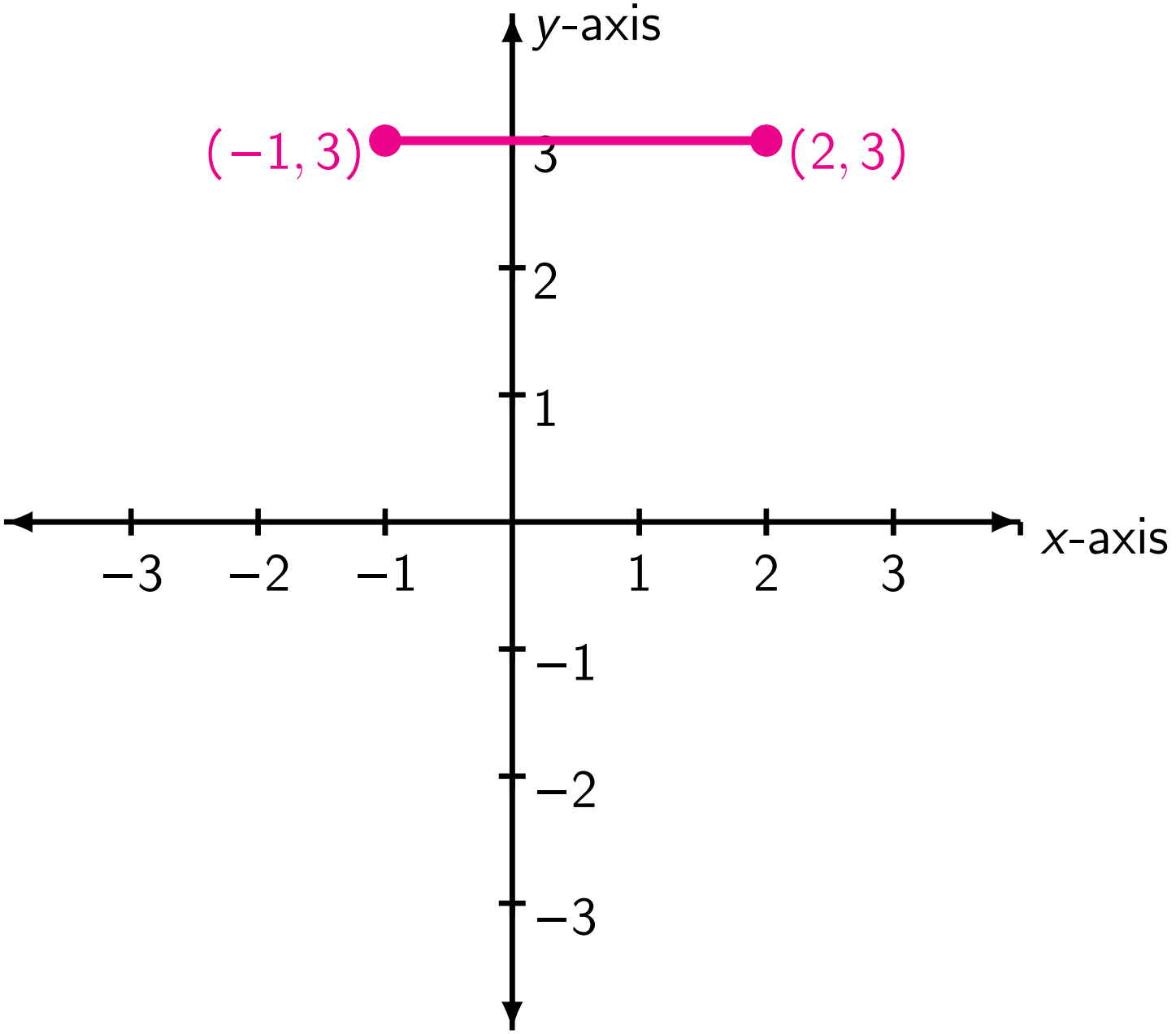
Relation $\Omega = \{(x, 3) | -1 \leq x \leq 2\}$



Relation $\Omega = \{(x, 3) | -1 \leq x \leq 2\}$



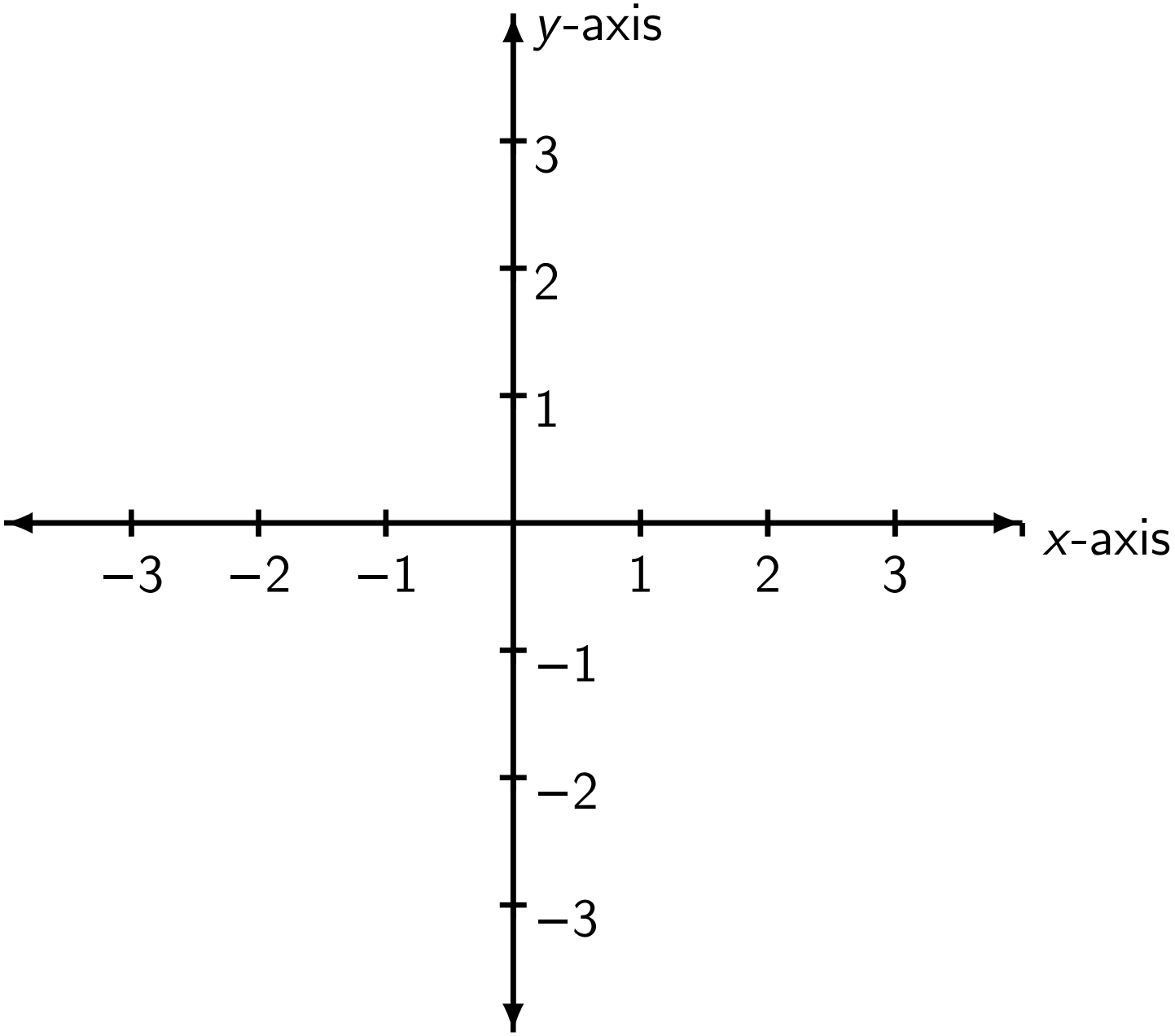
Relation $\Omega = \{(x, 3) | -1 \leq x \leq 2\}$



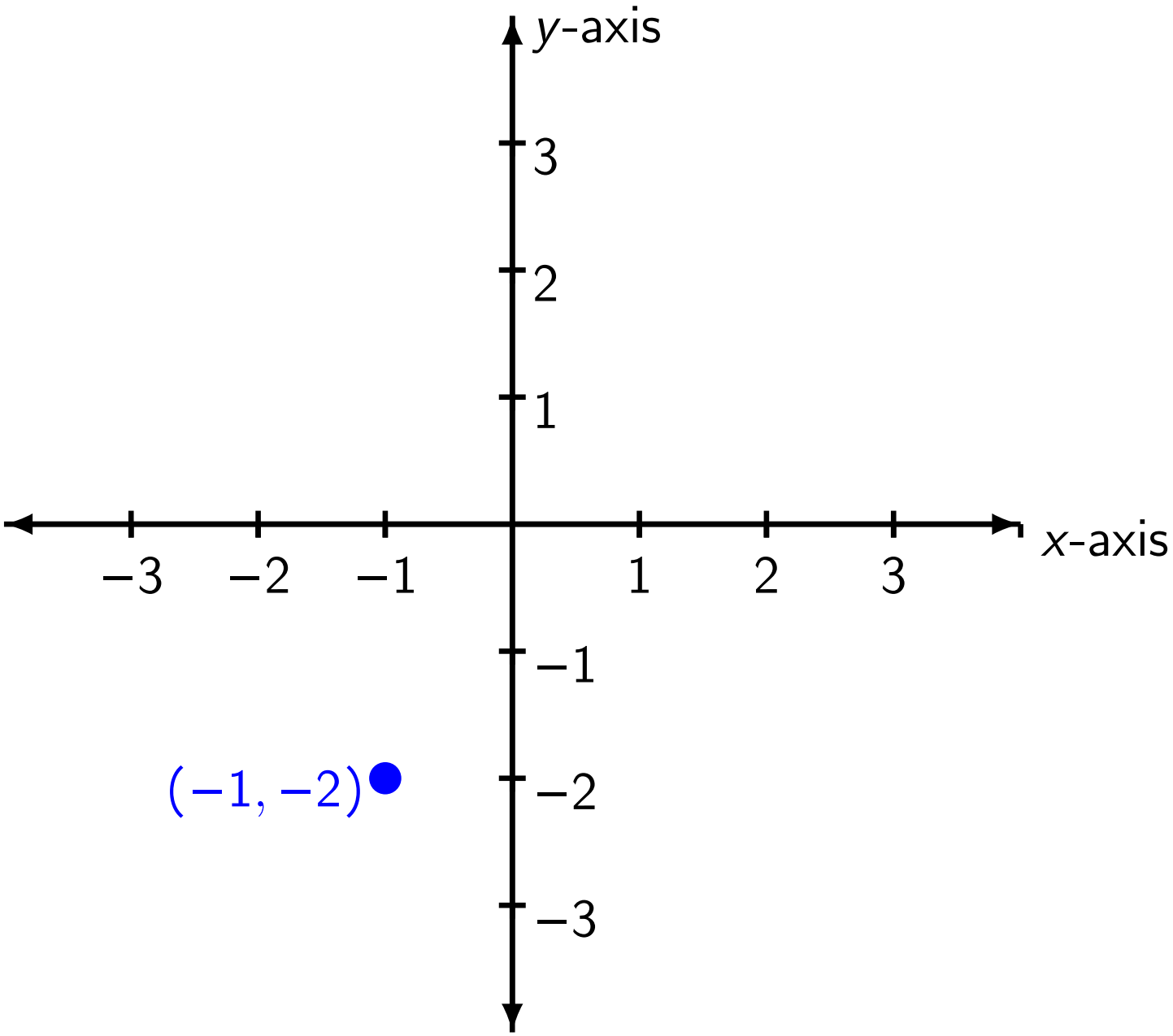
Examples of Relations

$\heartsuit = \{(-1, y) | -2 \leq y < 3\}$ is a relation

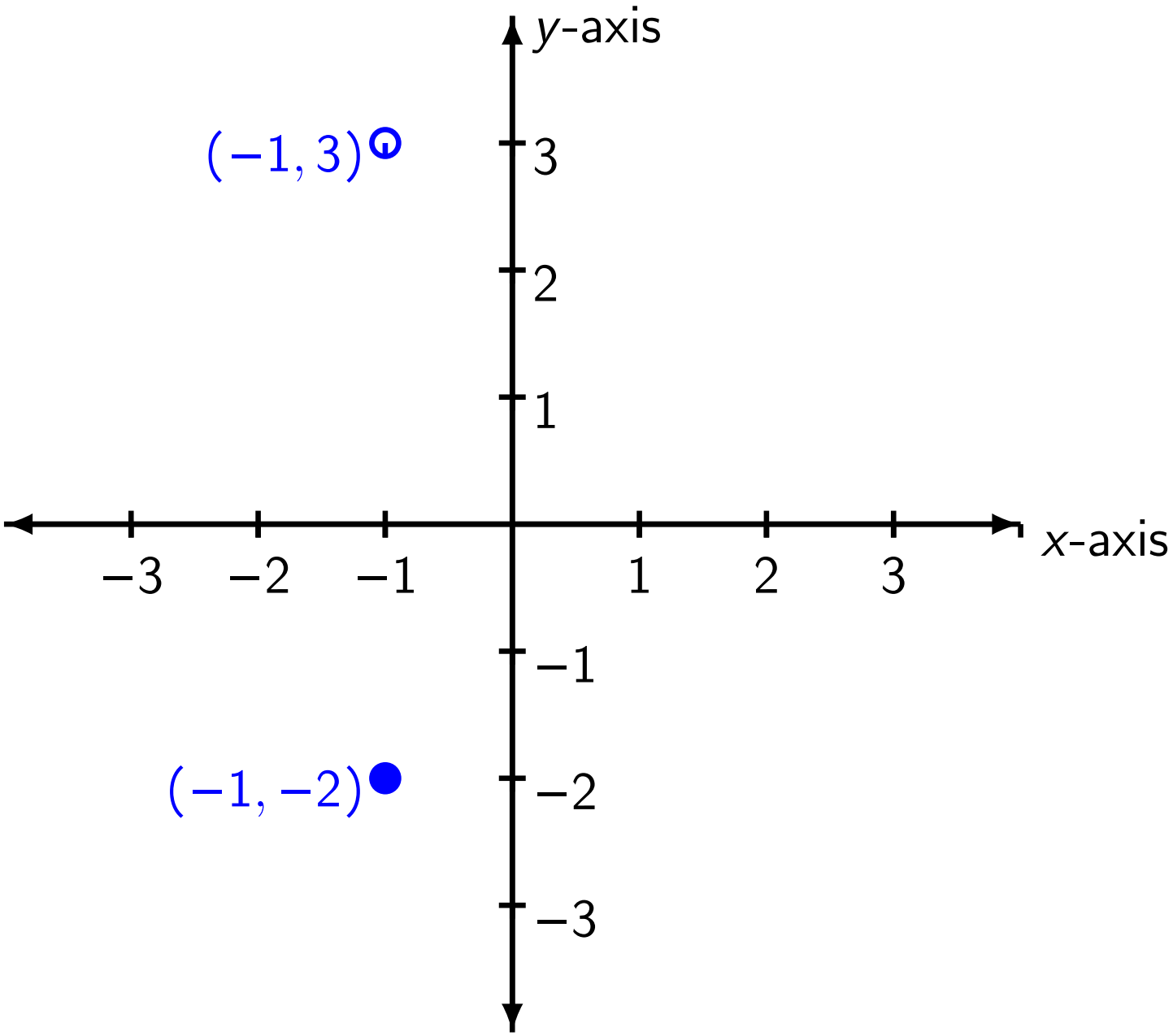
Relation $\heartsuit = \{(-1, y) | -2 \leq y < 3\}$



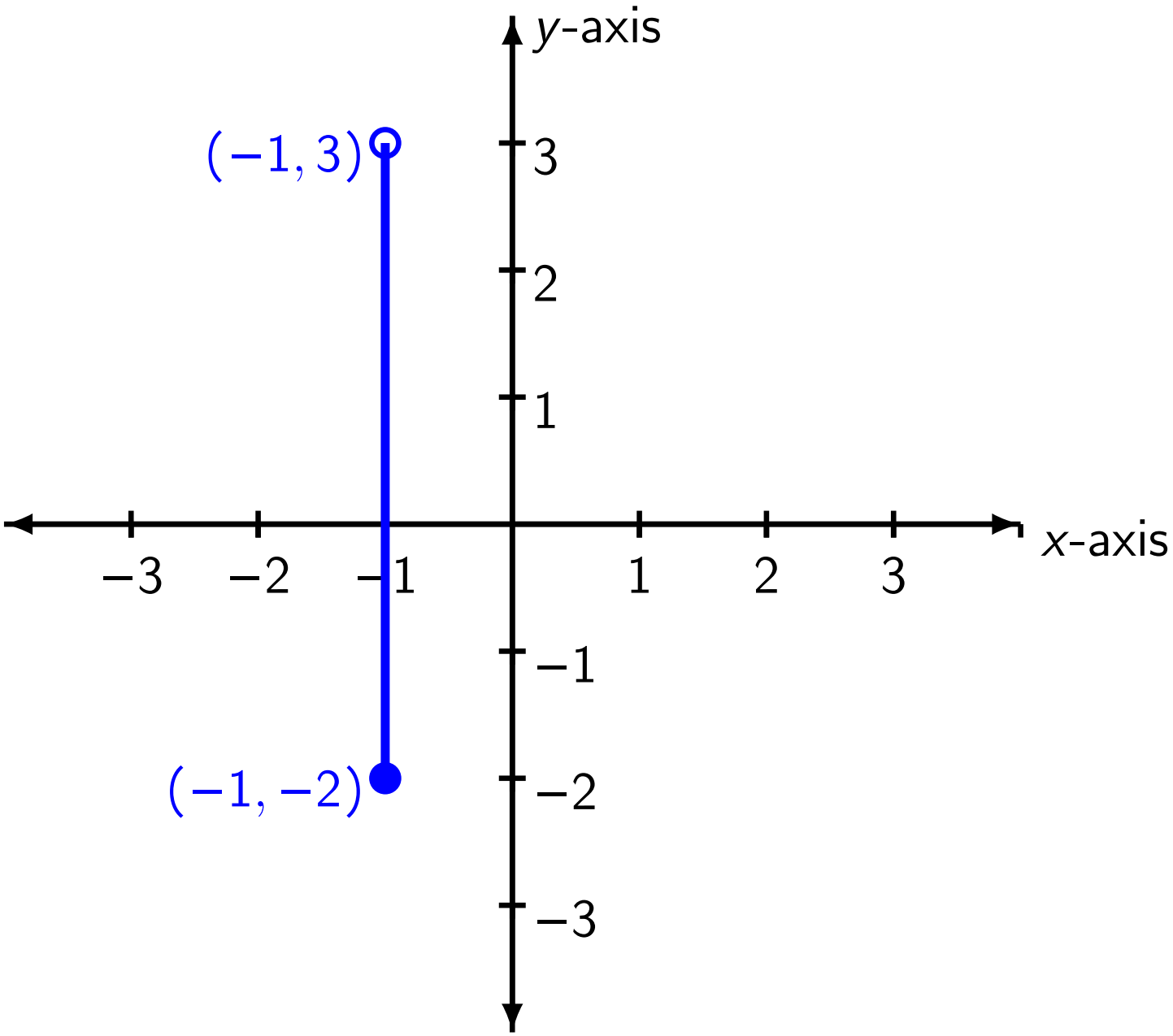
Relation $\heartsuit = \{(-1, y) | -2 \leq y < 3\}$



Relation $\heartsuit = \{(-1, y) | -2 \leq y < 3\}$



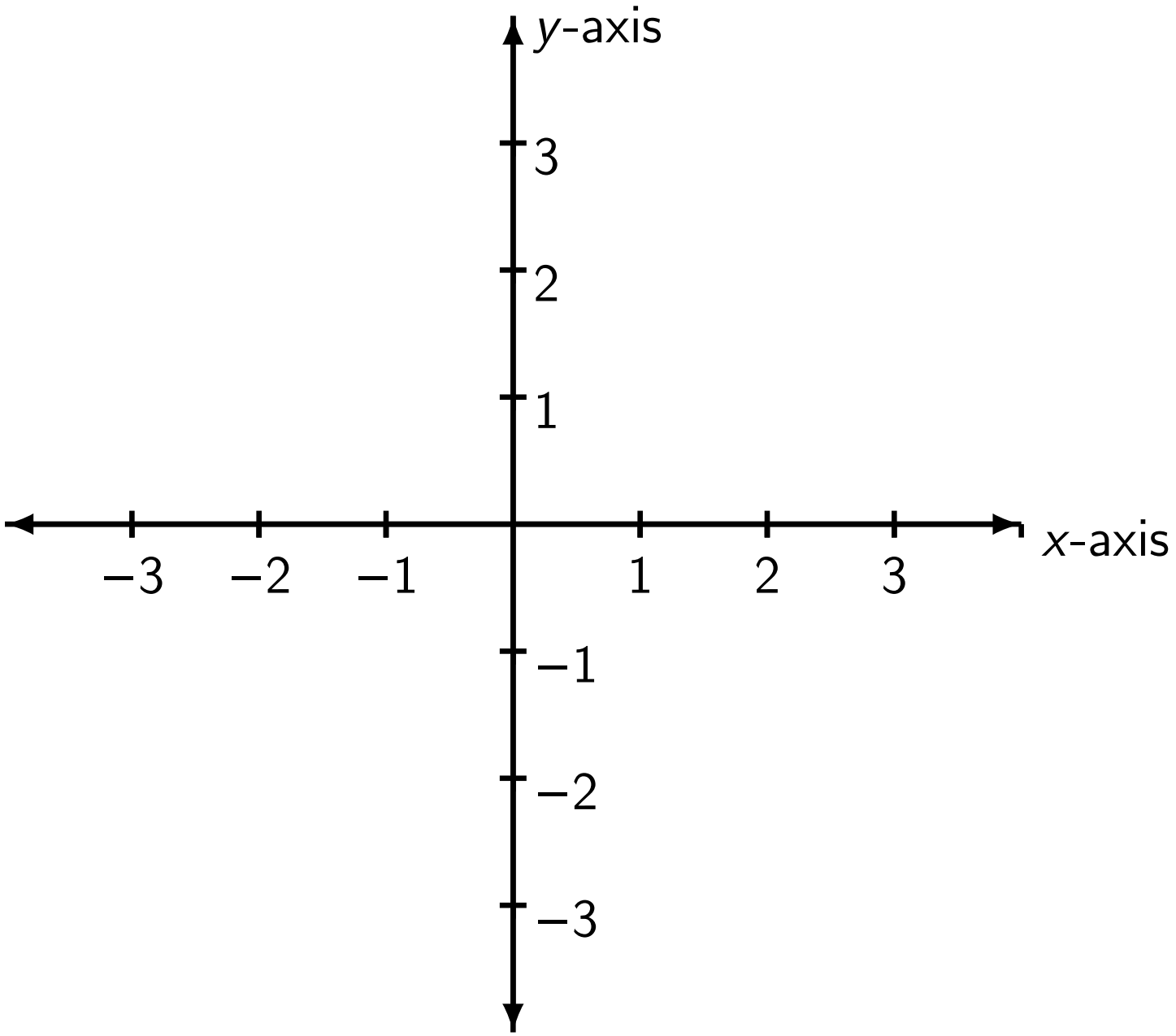
Relation $\heartsuit = \{(-1, y) | -2 \leq y < 3\}$



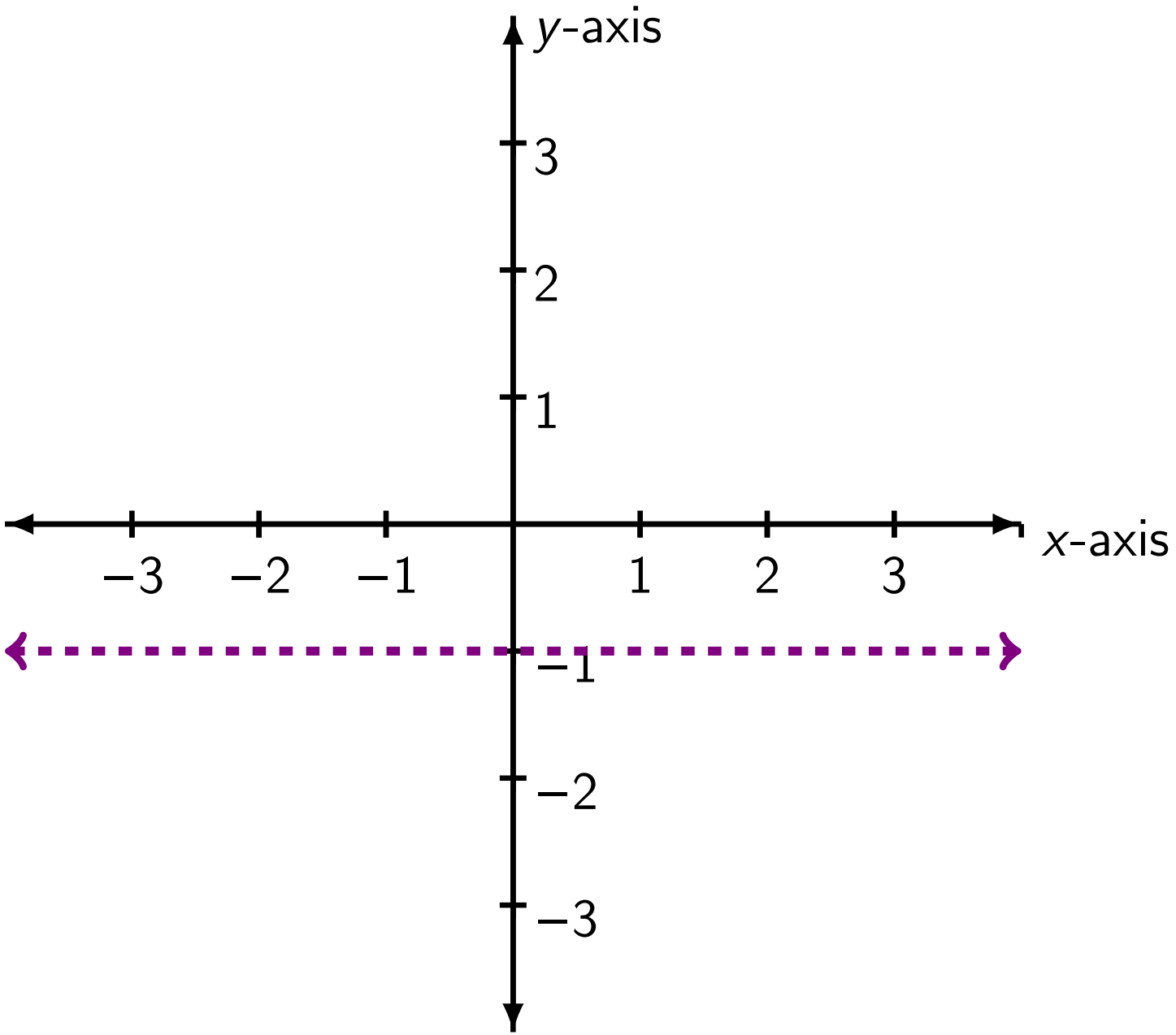
Examples of Relations

$\bigstar = \{(x, y) | -1 < y \leq 3\}$ is a relation

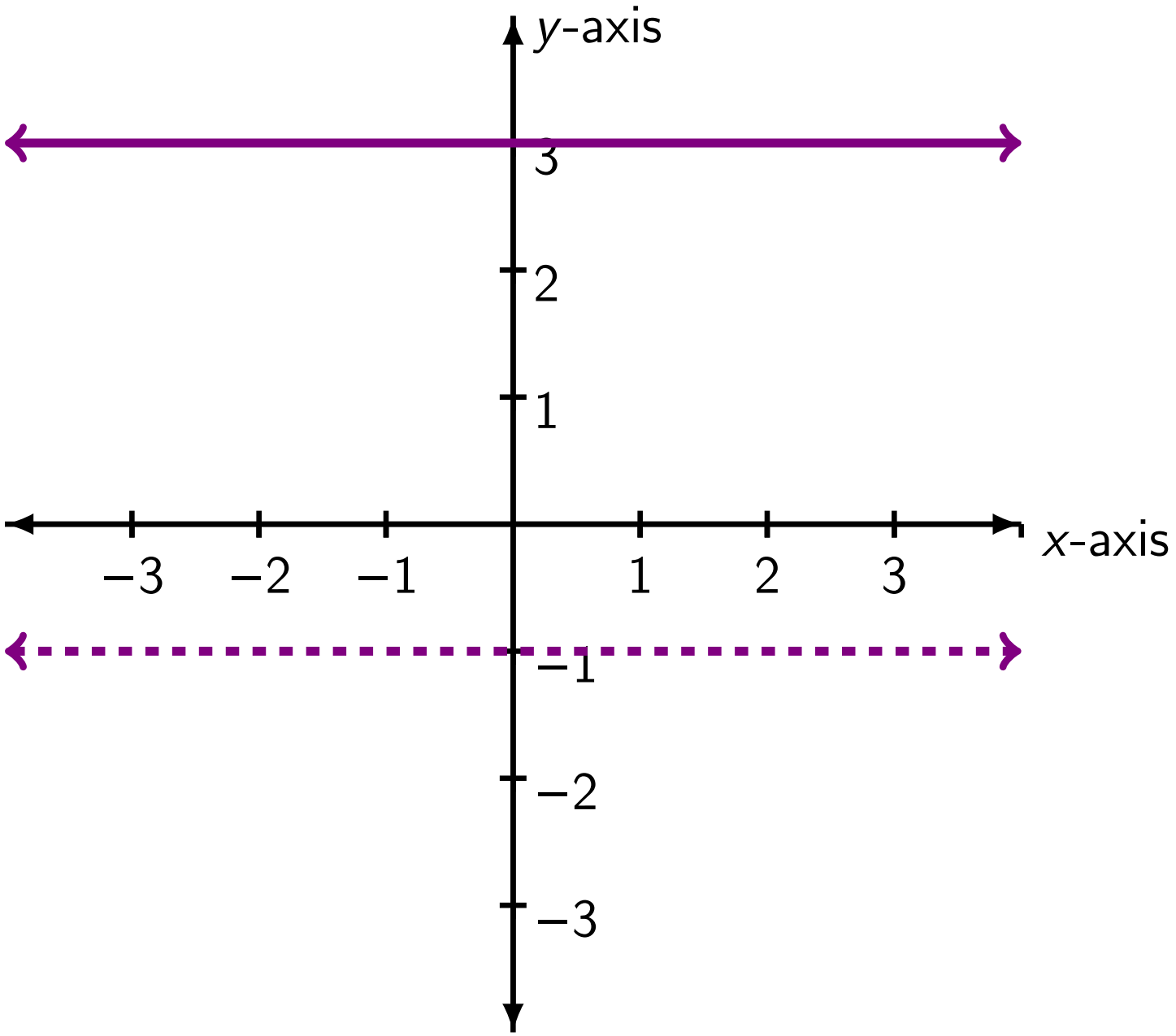
Relation $\bigstar = \{(x, y) | -1 < y \leq 3\}$



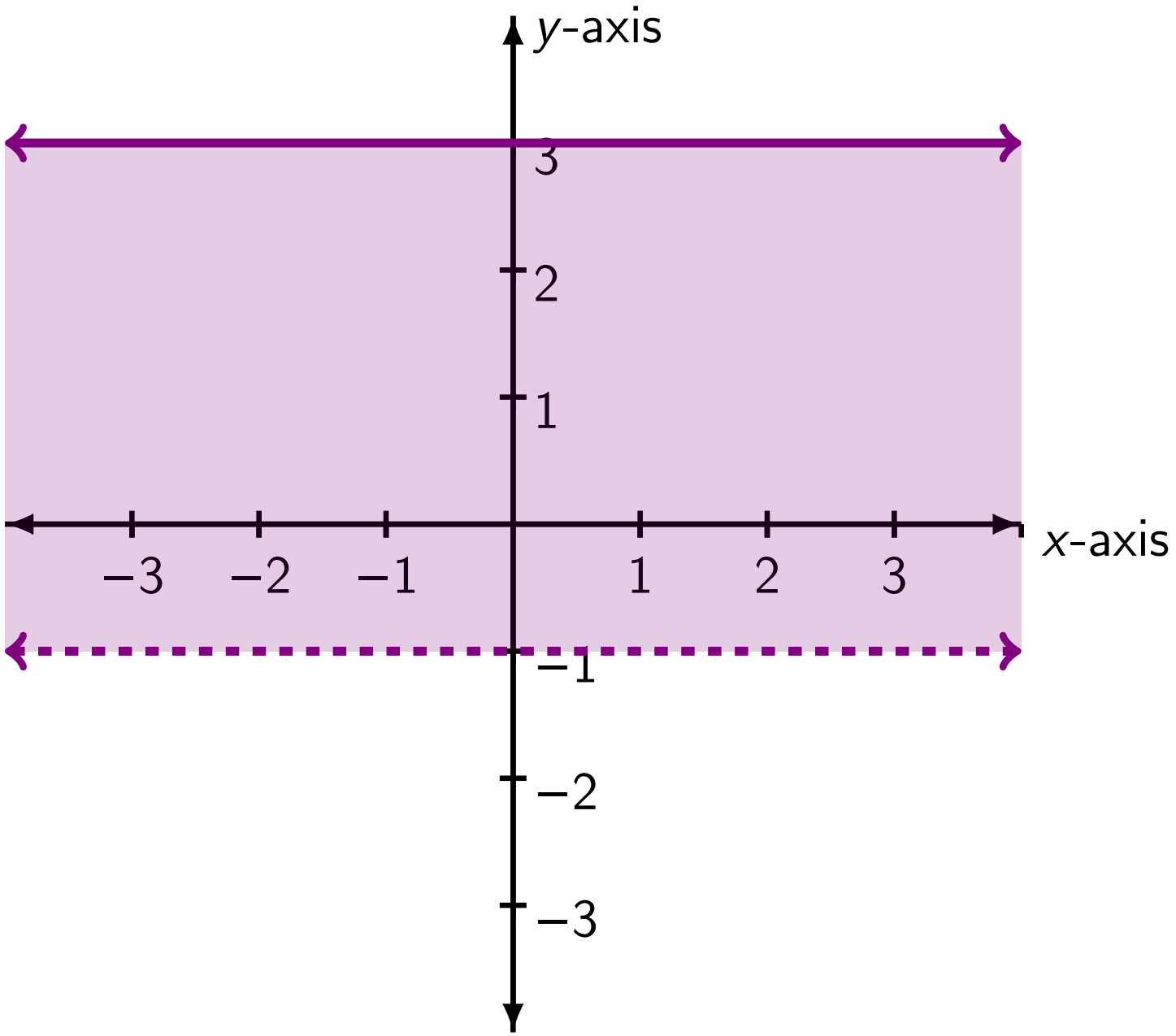
Relation $\bigstar = \{(x, y) | -1 < y \leq 3\}$



Relation $\bigstar = \{(x, y) | -1 < y \leq 3\}$



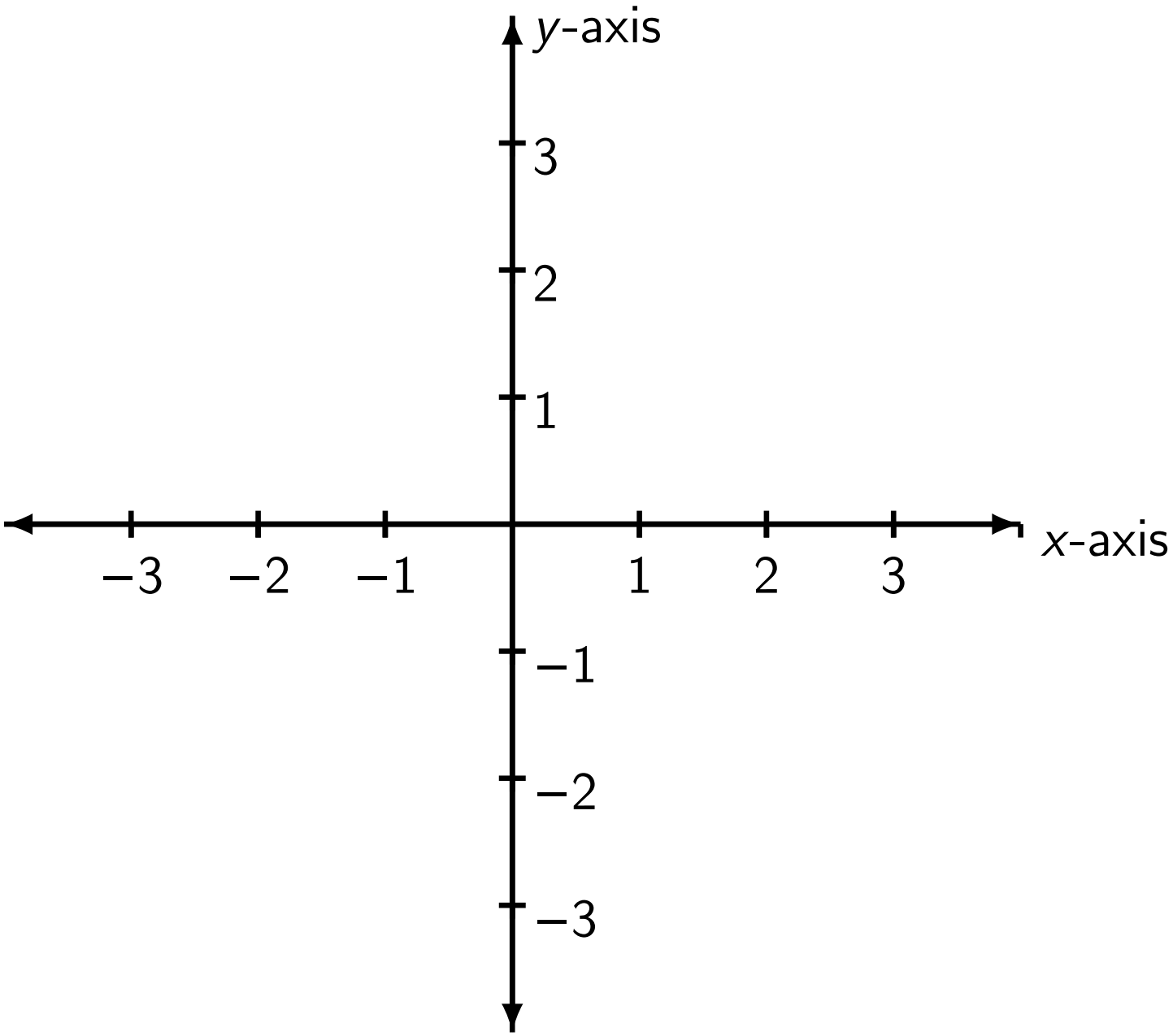
Relation $\bigstar = \{(x, y) | -1 < y \leq 3\}$



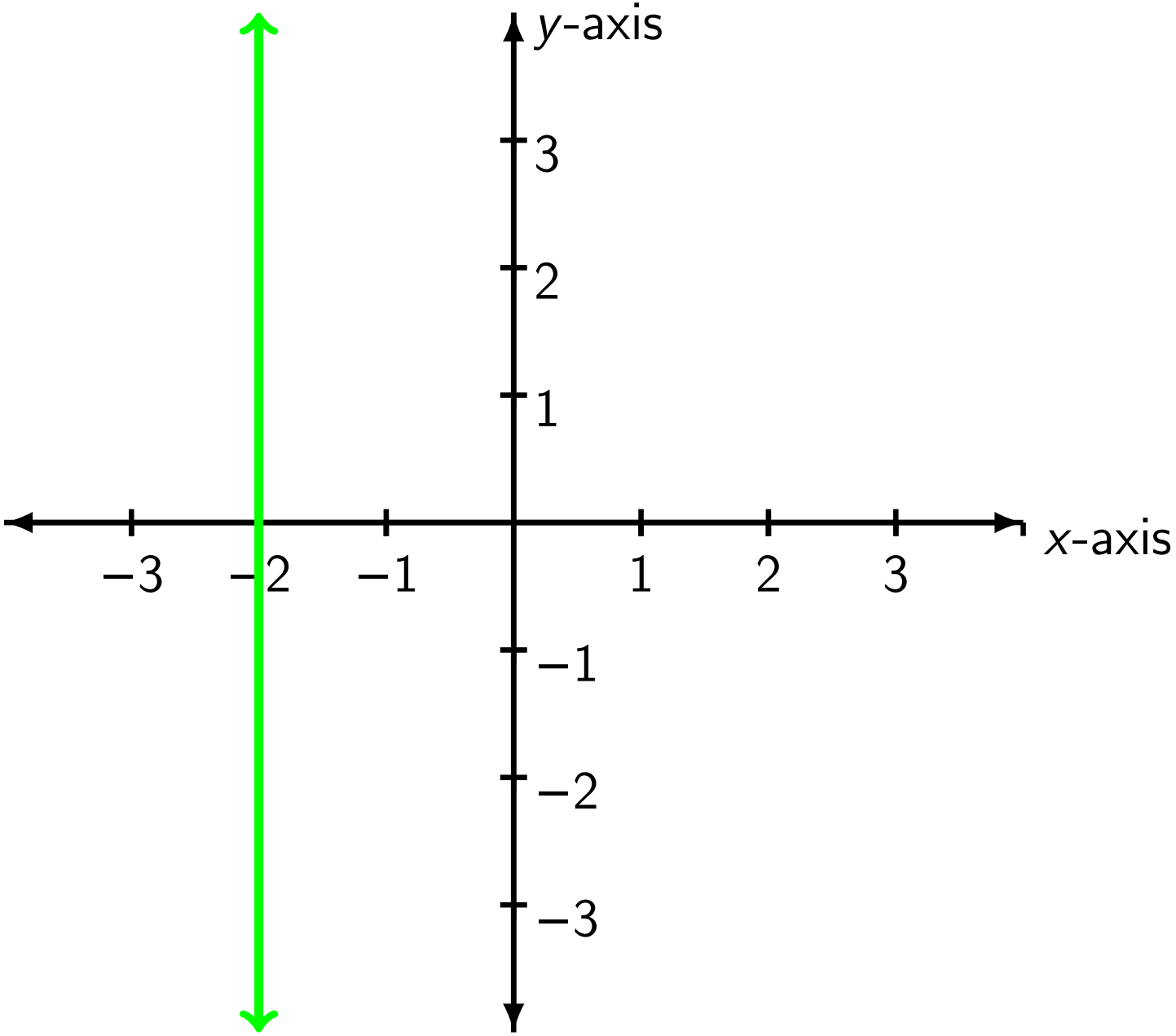
Examples of Relations

$\spadesuit = \{(x, y) | -2 \leq x < 2\}$ is a relation

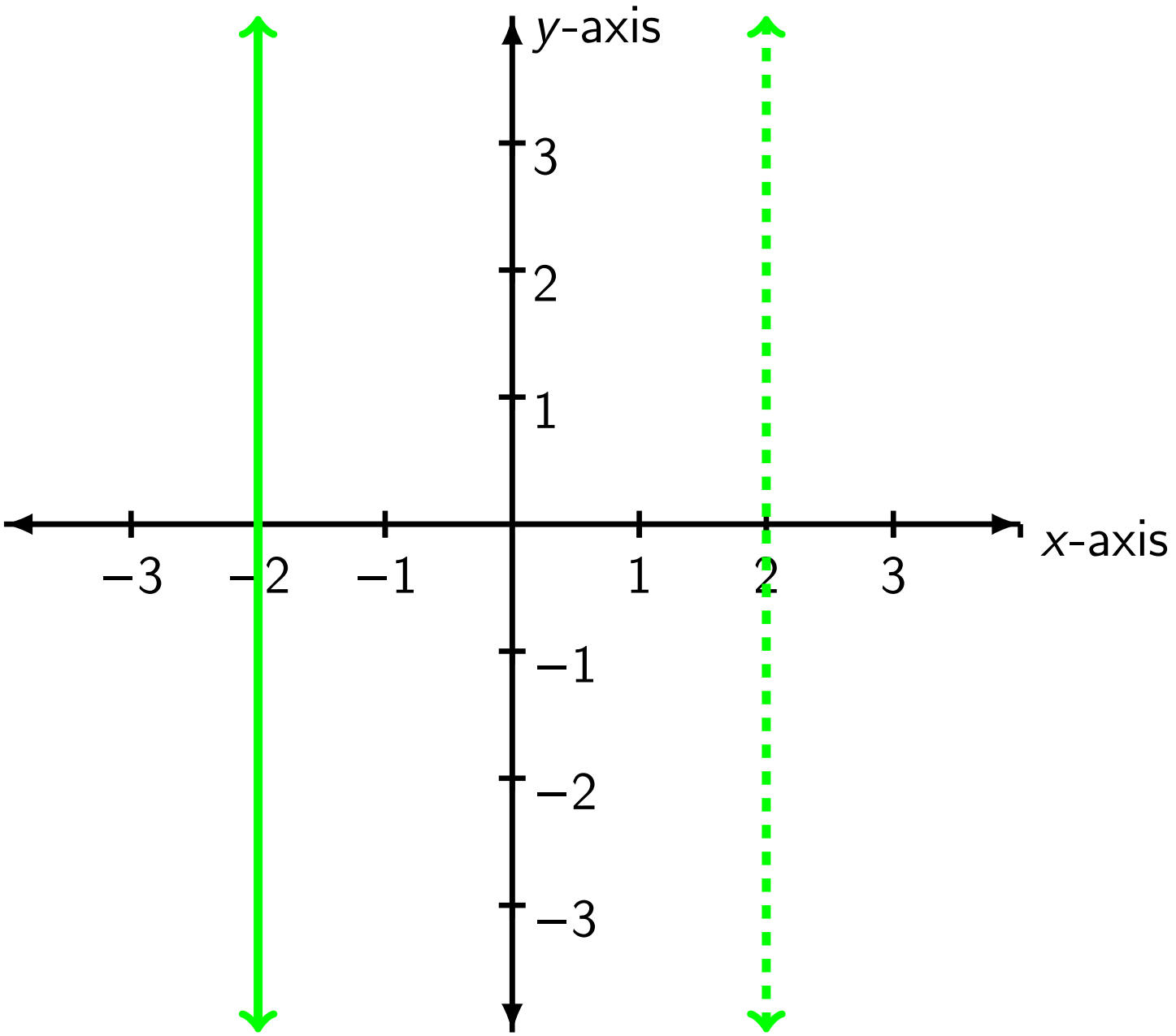
Relation $\spadesuit = \{(x, y) | -2 \leq x < 2\}$



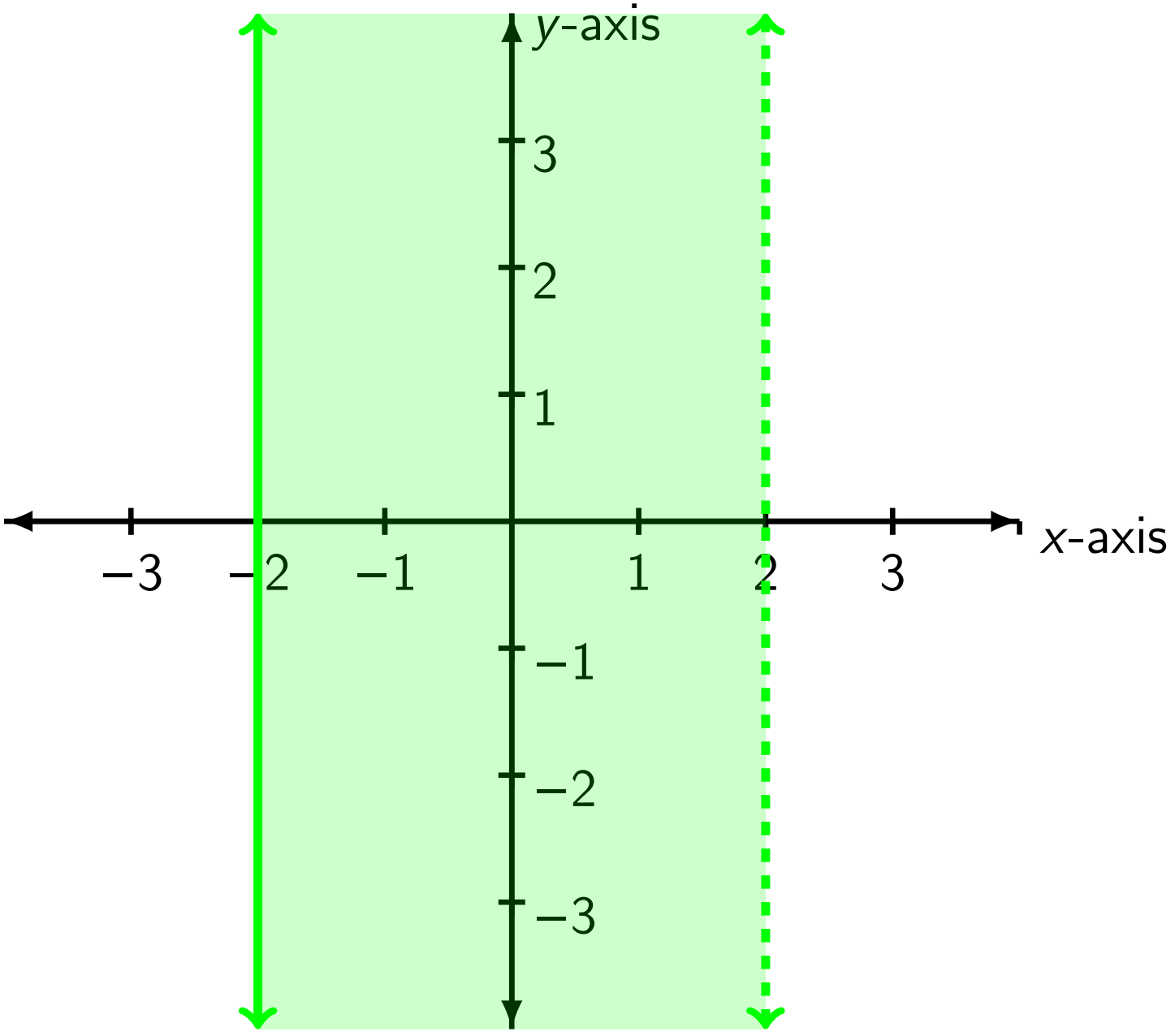
Relation ♠ $= \{(x, y) | -2 \leq x < 2\}$



Relation $\spadesuit = \{(x, y) | -2 \leq x < 2\}$



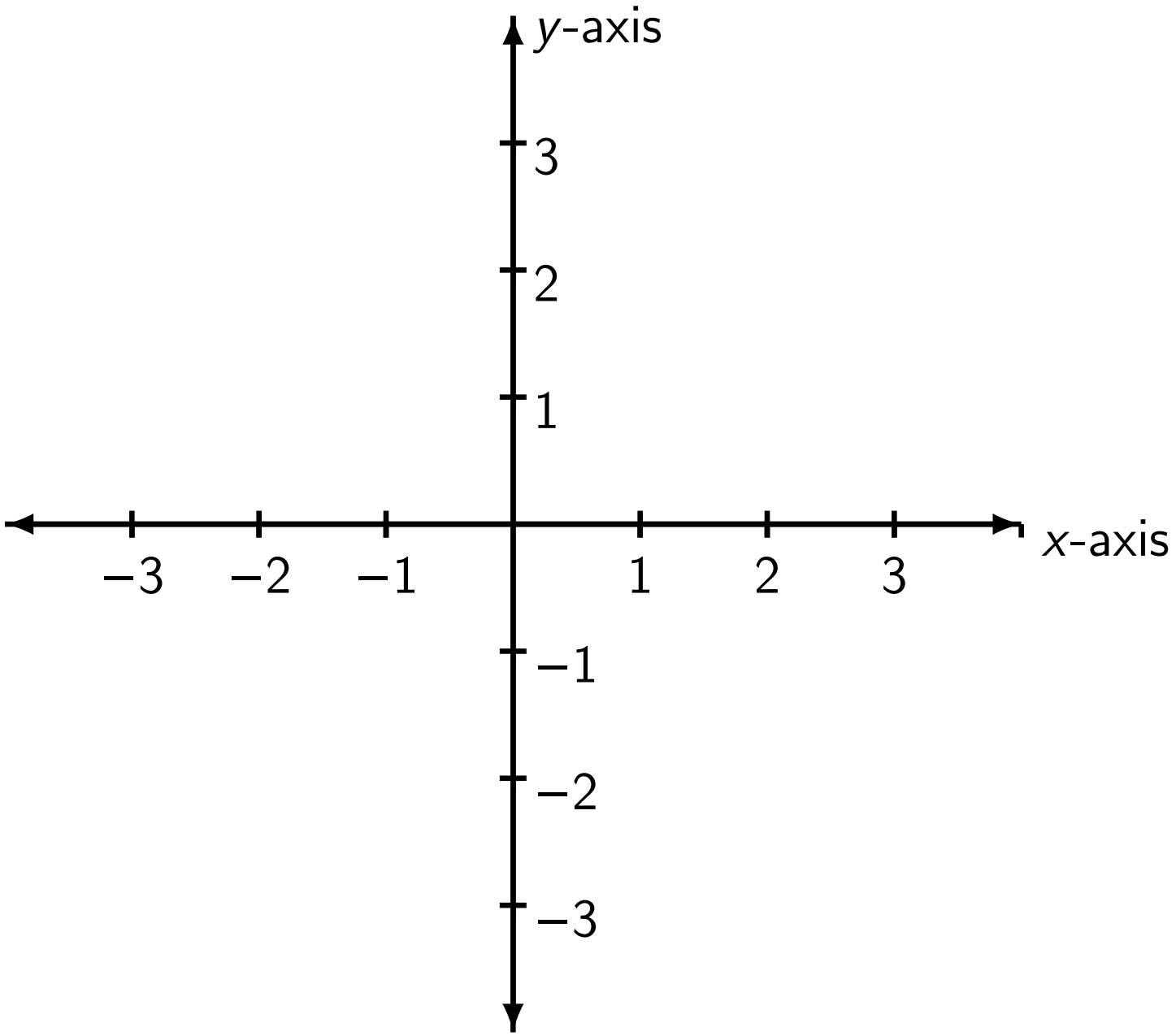
Relation $\spadesuit = \{(x, y) | -2 \leq x < 2\}$



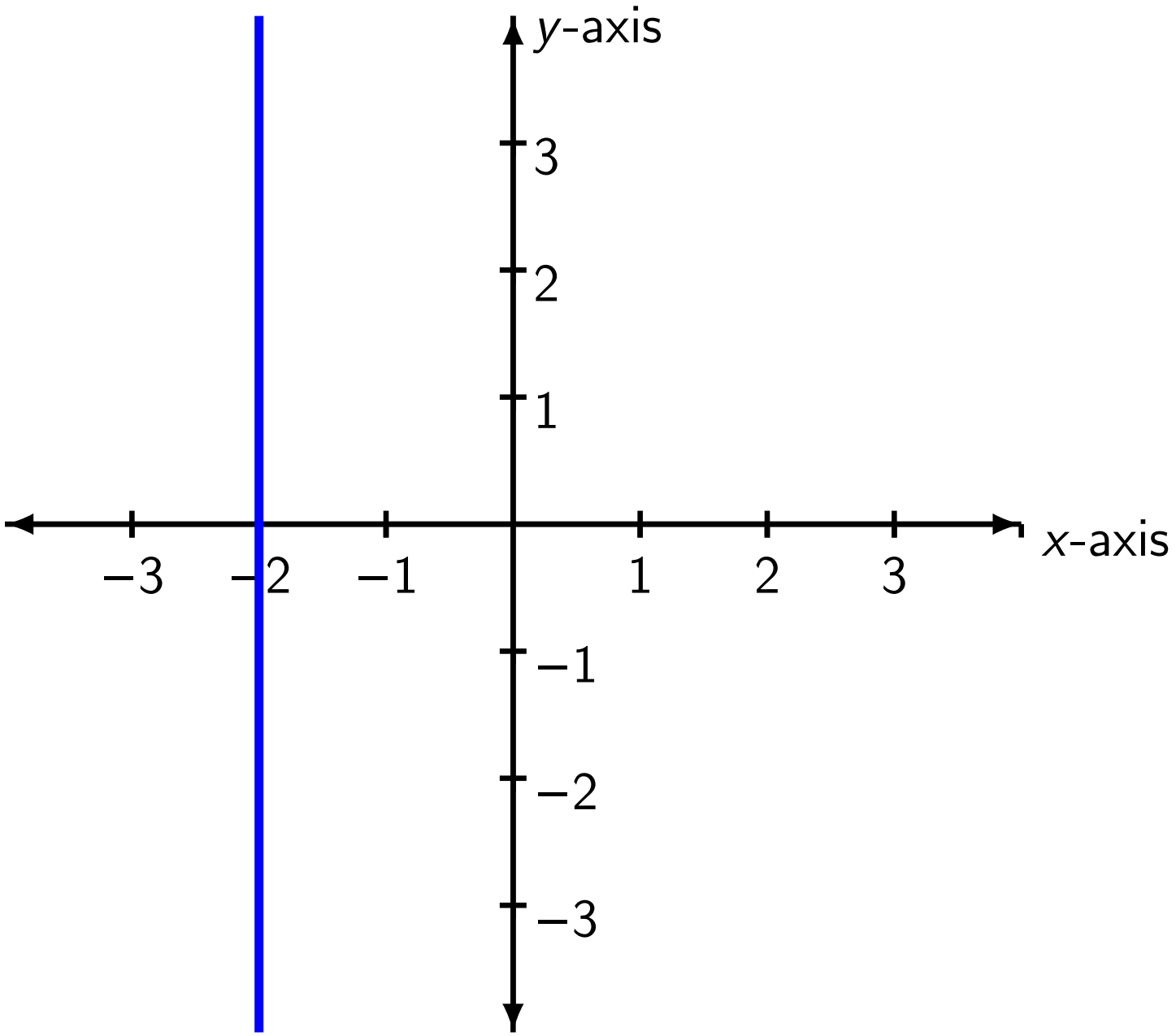
Examples of Relations

$\clubsuit = \{(x, y) | -2 \leq x < 2, -1 \leq y < 1\}$ is a relation

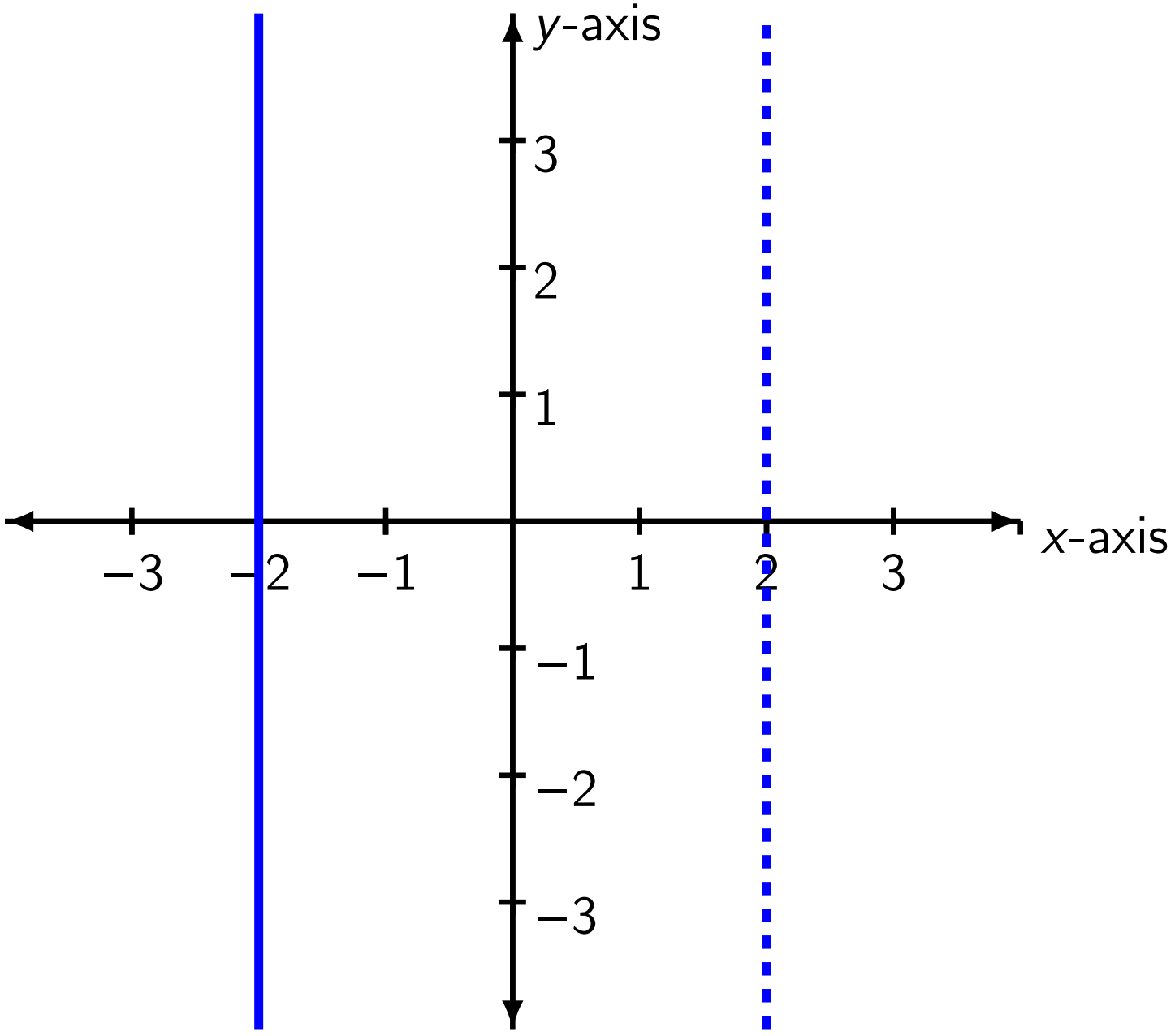
Relation $\clubsuit = \{(x, y) | -2 \leq x < 2, -1 \leq y < 1\}$



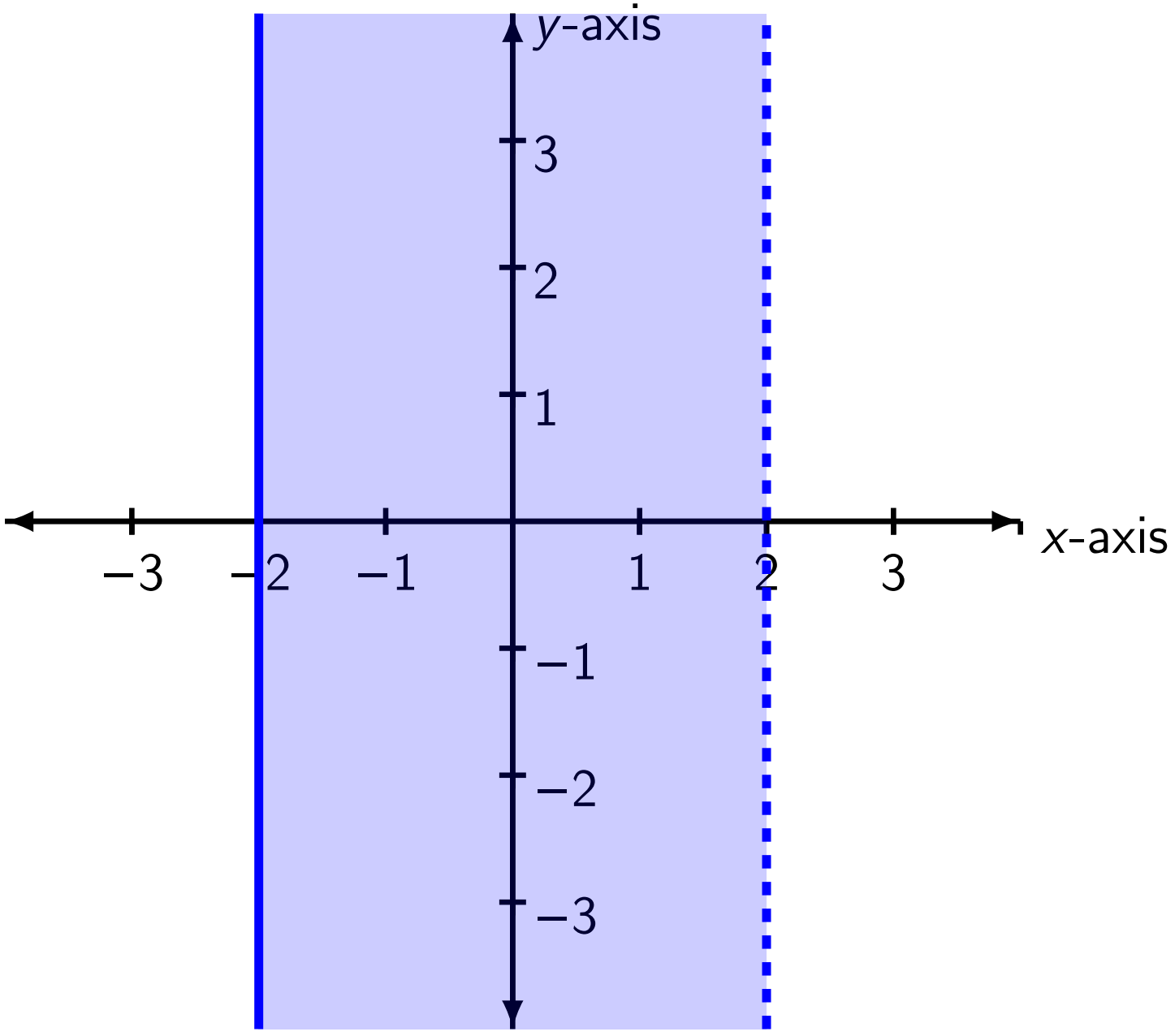
Relation $\clubsuit = \{(x, y) | -2 \leq x < 2, -1 \leq y < 1\}$



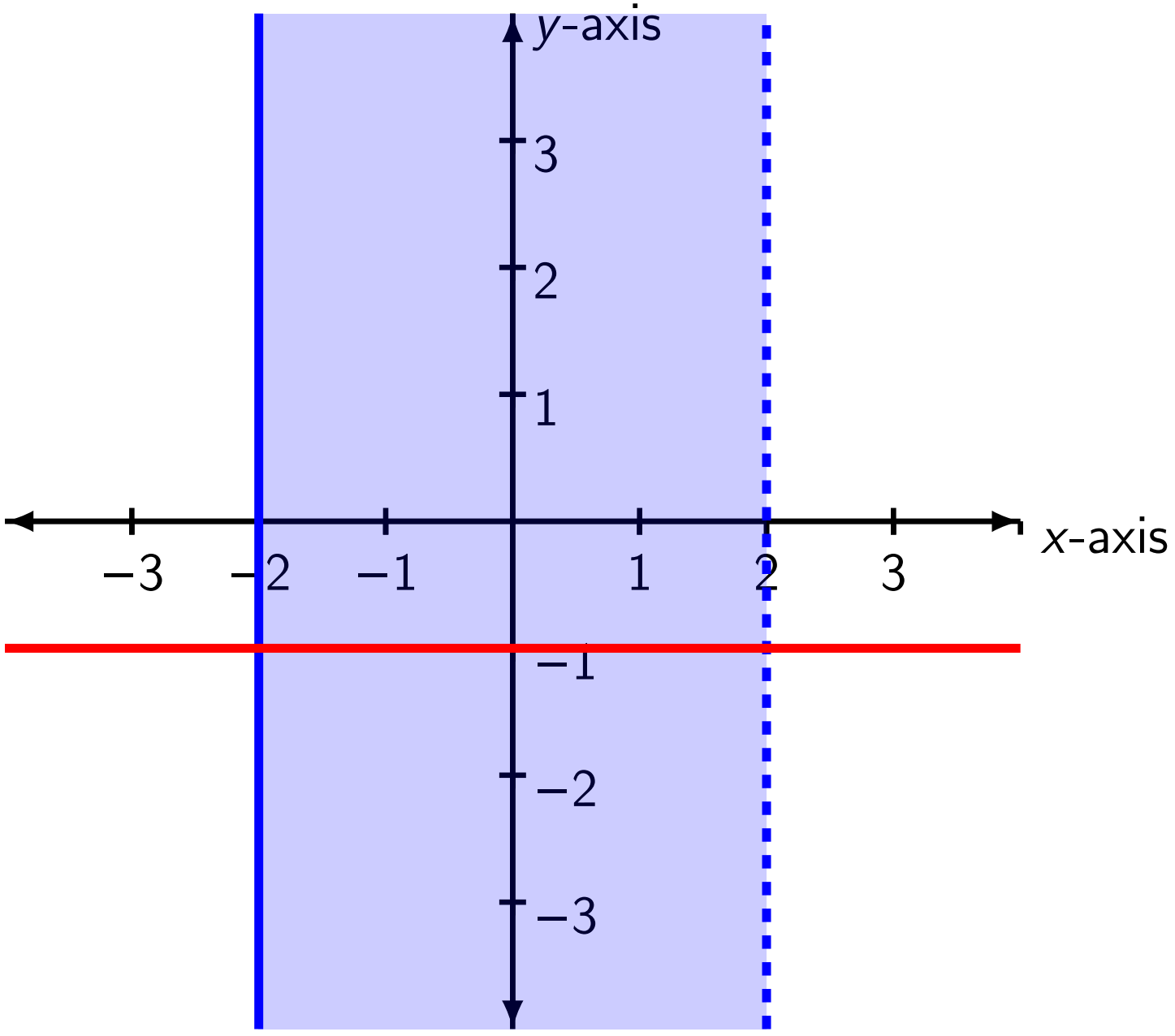
Relation $\clubsuit = \{(x, y) | -2 \leq x < 2, -1 \leq y < 1\}$



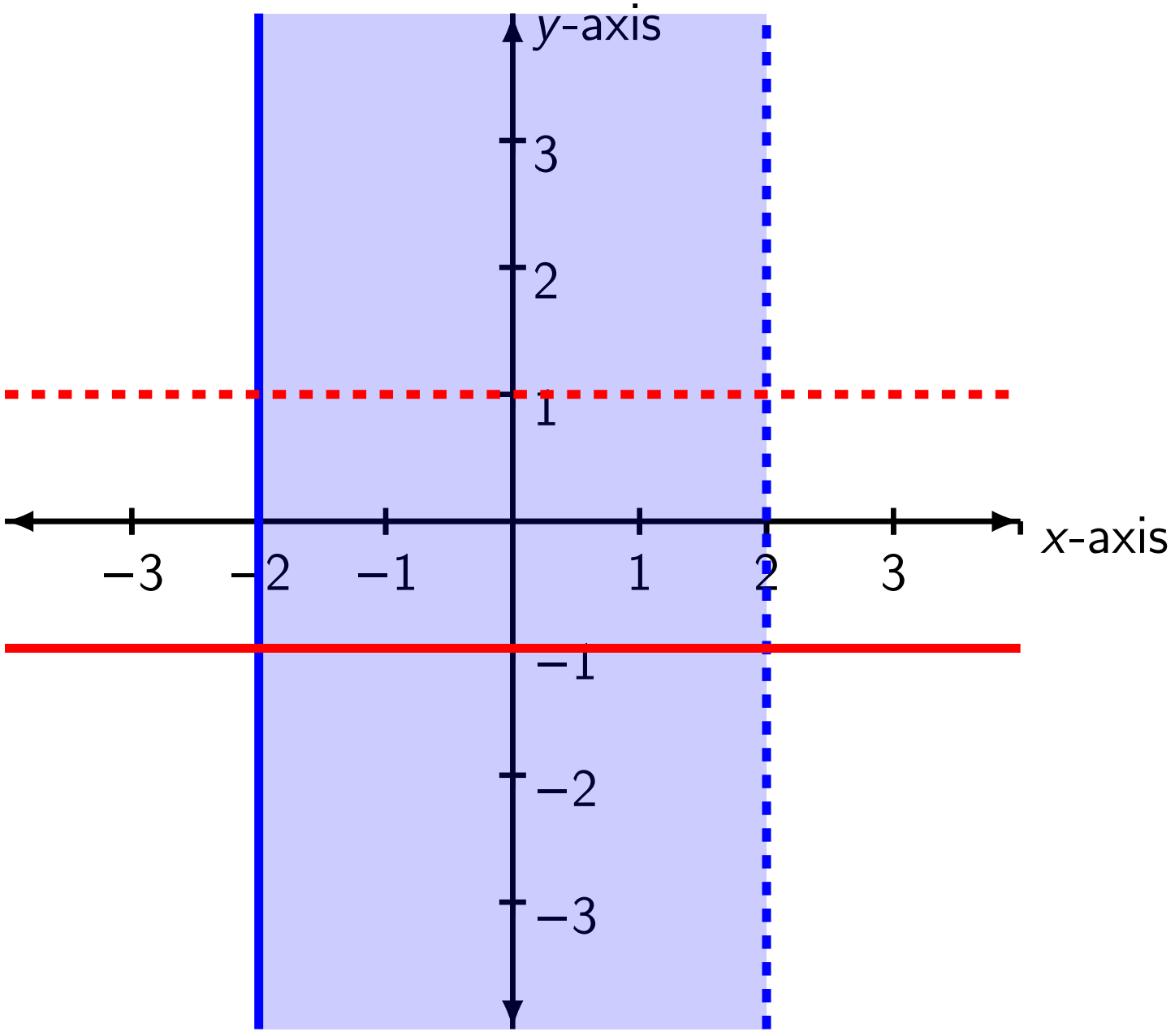
Relation $\clubsuit = \{(x, y) | -2 \leq x < 2, -1 \leq y < 1\}$



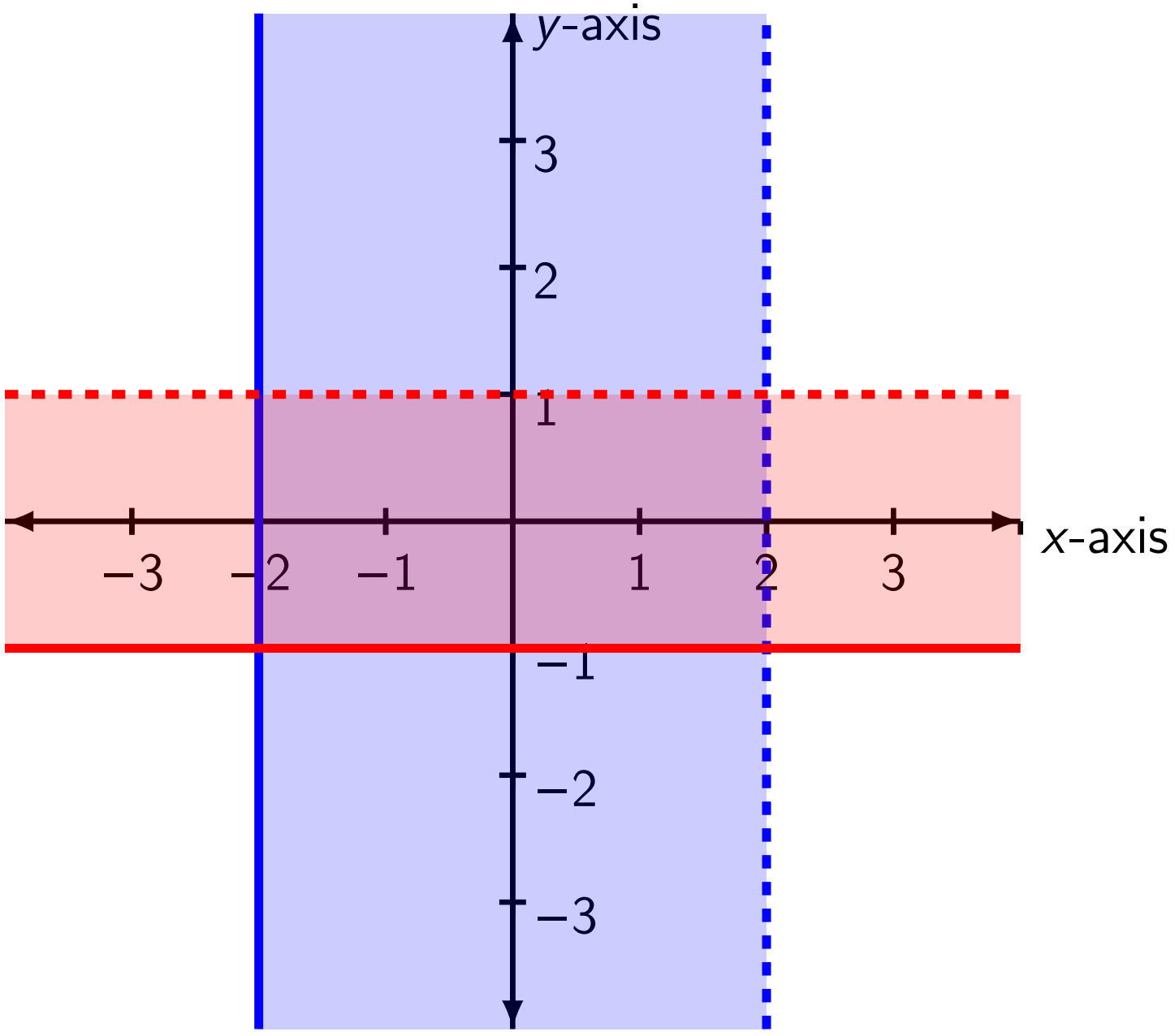
Relation $\clubsuit = \{(x, y) | -2 \leq x < 2, -1 \leq y < 1\}$



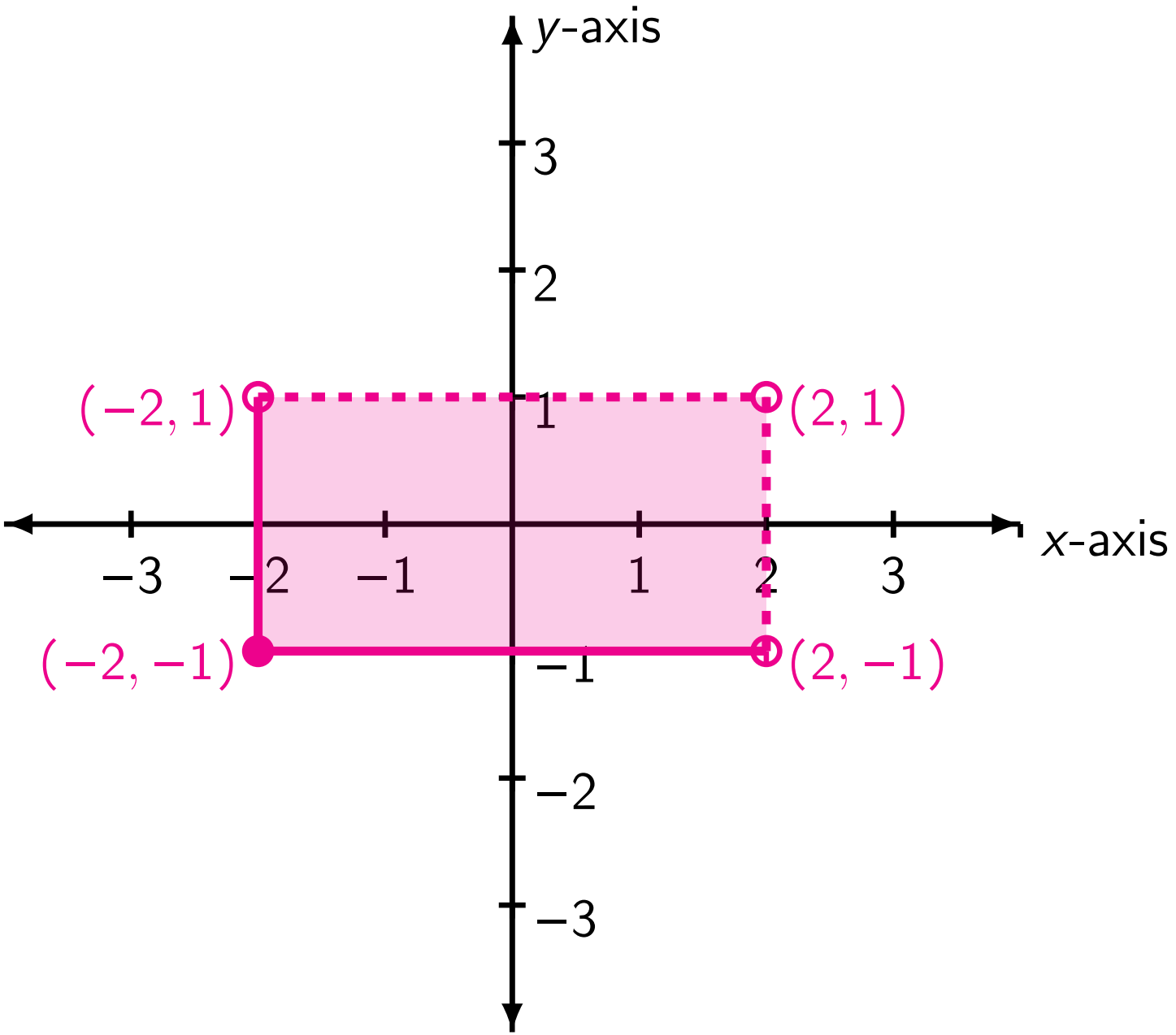
Relation $\clubsuit = \{(x, y) | -2 \leq x < 2, -1 \leq y < 1\}$



Relation $\clubsuit = \{(x, y) | -2 \leq x < 2, -1 \leq y < 1\}$



Relation $\clubsuit = \{(x, y) | -2 \leq x < 2, -1 \leq y < 1\}$



Important Note about Graphing

- The graph of the equation $x = a$ is a **vertical line** through $(a, 0)$.
- The graph of the equation $y = b$ is a **horizontal line** through $(0, b)$.

The Domain and Range of a Relation

A **relation** is a set of points in the plane or order pairs.

The Domain and Range of a Relation

A **relation** is a set of points in the plane or order pairs.

The **domain** of a relation is the set of first entries in each pair.

The Domain and Range of a Relation

A **relation** is a set of points in the plane or order pairs.

The **domain** of a relation is the set of first entries in each pair.

The **range** of a relation is the set of second entries in each pair.

Example Domain and Range of the Relation Γ

What is the domain of the relation

$\Gamma = \{(3, 3), (0, 3), (-2, 0), (2, -2)$?

Example Domain and Range of the Relation Γ

What is the domain of the relation

$\Gamma = \{(3,3),(0,3),(-2,0),(2,-2)$?

$\{3,0,-2,2\}$

Example Domain and Range of the Relation Γ

What is the domain of the relation

$\Gamma = \{(3,3),(0,3),(-2,0),(2,-2)$?

$\{3,0,-2,2\}$

What is the range of the relation

$\Gamma = \{(3,3),(0,3),(-2,0),(2,-2)$?

Example Domain and Range of the Relation Γ

What is the domain of the relation

$\Gamma = \{(3,3),(0,3),(-2,0),(2,-2)$?

$\{3,0,-2,2\}$

What is the range of the relation

$\Gamma = \{(3,3),(0,3),(-2,0),(2,-2)$?

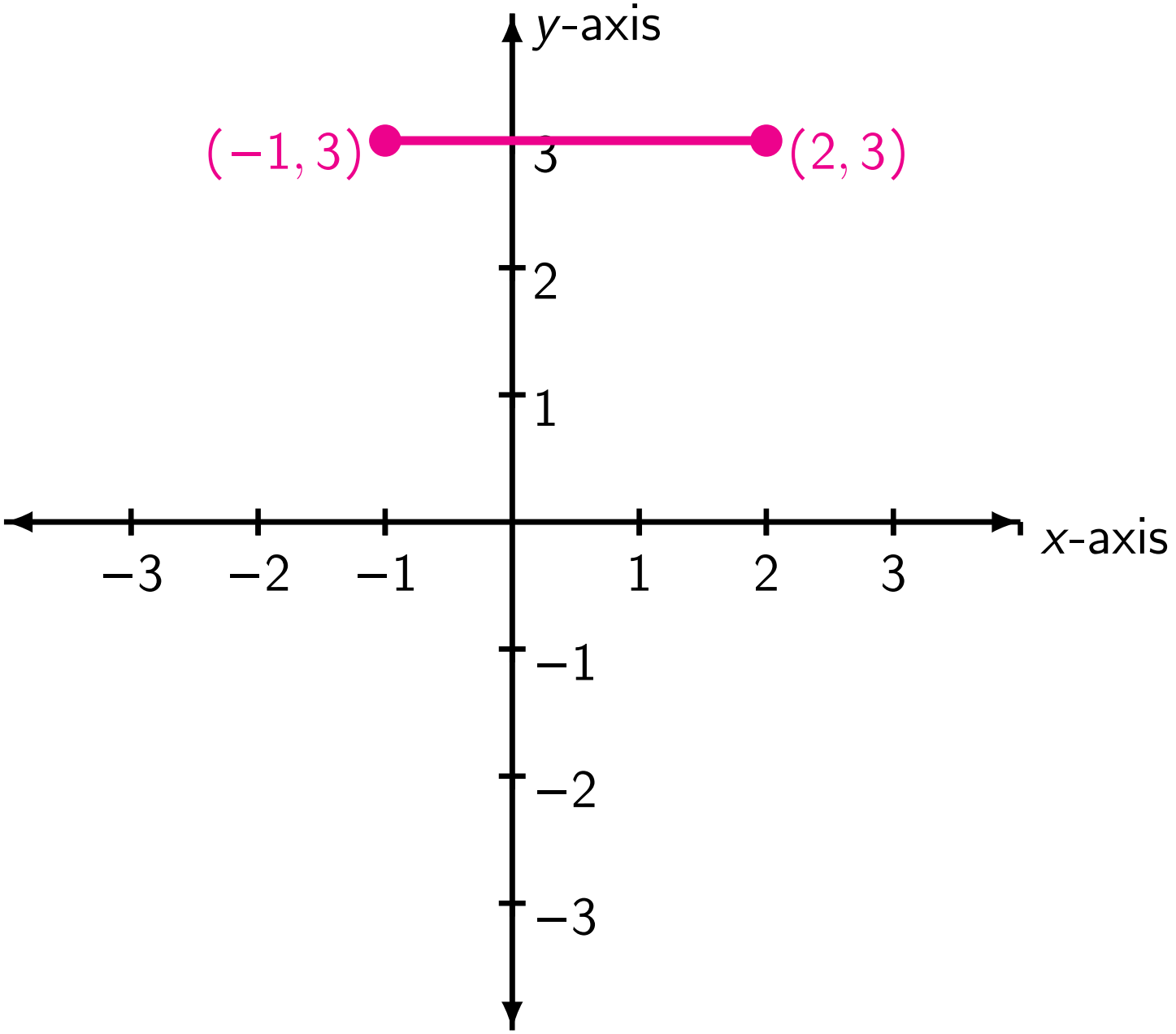
$\{3,0,-2\}$

Example Domain and Range of a Relation Ω

What is the domain and range of the relation

$\Omega = \{(x, 3) | -1 \leq x \leq 2\}$?

Relation $\Omega = \{(x, 3) | -1 \leq x \leq 2\}$



Example Domain and Range of a Relation Ω

What is the domain and range of the relation

$\Omega = \{(x, 3) | -1 \leq x \leq 2\}$?

Domain of $\Omega = \{x | -1 \leq x \leq 2\}$

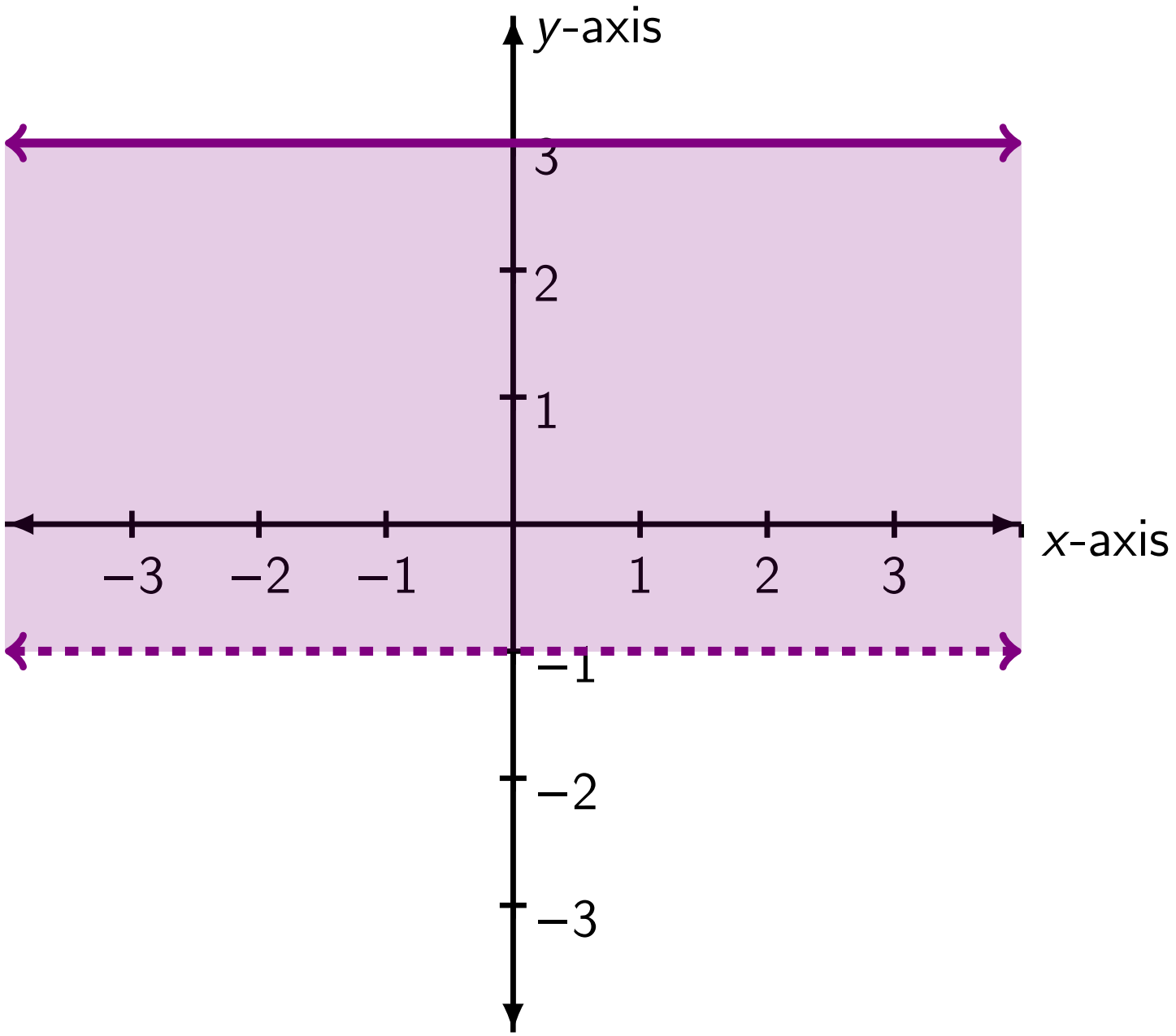
Range of $\Omega = \{3\}$.

Example Domain and Range of a Relation ★

What is the domain and range of the relation

$\bigstar = \{(x, y) | -1 < y \leq 3\}$?

Relation $\bigstar = \{(x, y) | -1 < y \leq 3\}$



Example Domain and Range of a Relation ★

What is the domain and range of the relation

$\bigstar = \{(x, y) | -1 < y \leq 3\}$?

Domain of $\bigstar = \{x | -\infty < x < \infty\}$

Range of $\bigstar = \{y | -1 < y \leq 3\}$

Functions

Definition of a Function

A relation in which each x-coordinate is matched with only one y-coordinate is said to describe y as a **function** of x.

Examples and NonExamples of Functions

Which of the following relations describe y as a function of x?

$$R_1 = \{(-2, 1), (1, 3), (1, 4), (3, -1)\}$$

$$R_2 = \{(-2, 1), (1, 3), (2, 3), (3, -1)\}$$

Examples and NonExamples of Functions

Which of the following relations describe y as a function of x?

$R_1 = \{(-2, 1), (1, 3), (1, 4), (3, -1)\}$

R_1 is NOT a function.

$R_2 = \{(-2, 1), (1, 3), (2, 3), (3, -1)\}$

R_2 is a function ☺.