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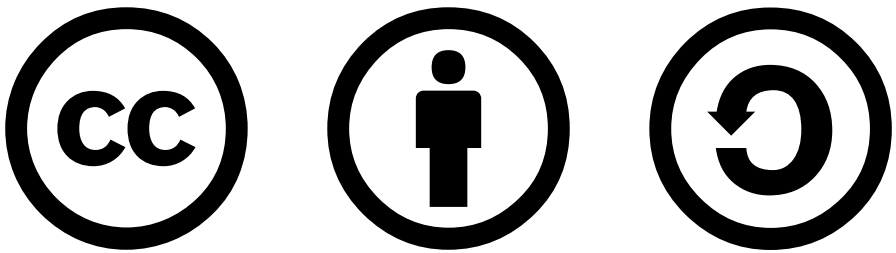

Algebra Lecture 3

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December 20, 2016

Warm Up Problems

1. Find the area of a square if a side has length $\heartsuit$.
2. Find the volume of a cube is an edge has length $3\spadesuit\diamondsuit$.

Warm Up Problems

1. Find the area of a square if a side has length $\heartsuit$. Solution: $\boxed{\heartsuit^2}$
2. Find the volume of a cube is an edge has length $3\spadesuit\diamondsuit$.
   Solution: $(3\spadesuit\diamondsuit)^3 = \boxed{27\spadesuit^3\diamondsuit^3}$

Exponents

Exponentiation

$$a^n = \overbrace{a * a * \cdots * a}^{n}$$

Exponents

Properties of Exponents

$$a^0 \equiv 1$$

$$a^n a^m = a^{n+m}$$

$$a^n a^{-m} = a^{n+-m}$$

$$a^{-1} \equiv \frac{1}{a}$$

$$a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$$

Properties of Exponents

Properties of a^n

$a^n a^m = a^{n+m}$

Properties of Exponents

Properties of a^n

$$a^n a^m = a^{n+m}$$

Why?

Properties of Exponents

Properties of a^n

$$a^n a^m = a^{n+m}$$

Why?

$$a^n * a^m = \overbrace{a * a * \cdots * a}^{n} * \overbrace{a * a * \cdots * a}^{m} = a^{n+m}$$

Properties of Exponents

Properties of a^n

$$a^n a^m = a^{n+m}$$

Why?

$$a^n * a^m = \overbrace{a * a * \cdots * a}^{n} * \overbrace{a * a * \cdots * a}^{m} = a^{n+m}$$

$$3^2 3^3 = \overbrace{3 * 3}^{2} * \overbrace{3 * 3 * 3}^{3} = 3^5 = 3^{2+3}$$

Examples

Properties of Exponents Examples

Example

$$\diamondsuit^4 \heartsuit^2 \alpha \diamondsuit^3$$

$$= \diamondsuit^4 \heartsuit^2 \diamondsuit^3 \alpha \quad \text{(commutativity)}$$

$$= \diamondsuit^4 \diamondsuit^3 \heartsuit^2 \alpha \quad \text{(commutativity)}$$

$$= \diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit\diamondsuit\heartsuit^2 \alpha$$

$$= \boxed{\diamondsuit^7 \heartsuit^2 \alpha}$$

α -alpha

Properties of Exponents

Properties of $\left(\frac{a}{b}\right)^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Properties of Exponents

Properties of $\left(\frac{a}{b}\right)^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Why?

Properties of Exponents

Properties of $\left(\frac{a}{b}\right)^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Why?

$$\left(\frac{a}{b}\right)^n = \overbrace{\left(\frac{a}{b}\right) * \cdots * \left(\frac{a}{b}\right)}^{n} = \frac{\overbrace{a * \cdots * a}^{n}}{\underbrace{b * \cdots * b}_{n}} = \frac{a^n}{b^n}$$

Properties of Exponents

Properties of $\left(\frac{a}{b}\right)^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Why?

$$\left(\frac{a}{b}\right)^n = \overbrace{\left(\frac{a}{b}\right) * \cdots * \left(\frac{a}{b}\right)}^{n} = \frac{\overbrace{a * \cdots * a}^{n}}{\underbrace{b * \cdots * b}_{n}} = \frac{a^n}{b^n}$$

$$\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right) * \left(\frac{3}{4}\right) = \frac{3 * 3}{4 * 4} = \boxed{\frac{3^2}{4^2}}$$

Properties of Exponents

Definition a^{-n}

$$a^{-n} \equiv \frac{1}{a^n} \qquad \& \qquad a^m a^{-n} = a^{m+(-n)}$$

Properties of Exponents

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Why?

Properties of Exponents

Definition a^{-n}

$$a^{-n} \equiv \frac{1}{a^n} \qquad \& \qquad a^m a^{-n} = a^{m+(-n)}$$

Why?

$$a^m * a^{-n} = \overbrace{a * a * \cdots * a}^{m} * \frac{1}{\underbrace{a * a * \cdots * a}_{n}}$$

Properties of Exponents

Definition a^{-n}

$$a^{-n} \equiv \frac{1}{a^n} \qquad \& \qquad a^m a^{-n} = a^{m+(-n)}$$

Why?

$$a^m * a^{-n} = \overbrace{a * a * \cdots * a}^{m} * \frac{1}{\underbrace{a * a * \cdots * a}_{n}}$$

$$3^2 3^{-4} = \overbrace{3 * 3}^{2} * \frac{1}{\underbrace{3 * 3 * 3 * 3}_{4}} = \frac{1}{3 * 3} = \frac{1}{3^2} = \boxed{3^{-2}}$$

Examples

$$\clubsuit^4 \heartsuit \clubsuit^{-2}$$

$$= \heartsuit \clubsuit^4 \clubsuit^{-2} \quad \text{(commutativity)}$$

$$= \heartsuit \clubsuit^4 \frac{1}{\clubsuit^2} \quad \text{(definition)}$$

$$= \heartsuit \clubsuit \clubsuit \clubsuit \clubsuit \frac{1}{\clubsuit \clubsuit}$$

$$= \heartsuit \clubsuit \clubsuit \cancel{\clubsuit} \cancel{\clubsuit} \frac{1}{\cancel{\clubsuit} \cancel{\clubsuit}}$$

$$= \heartsuit \clubsuit \clubsuit$$

$$= \boxed{\heartsuit \clubsuit^2}$$

Properties of Exponents Exercise for at Home

At Home Exercise

Prove that $\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$. (Hint: write out both sides of the equations using the other definitions and properties of exponents.)

Properties of Exponents Example

Problem

Simplify the following expression:

$$\left(\frac{10}{3}\right)^{-2}$$

Properties of Exponents Example

$$\left(\frac{10}{3}\right)^{-2}$$

Properties of Exponents Example

$$\left(\frac{10}{3}\right)^{-2}$$

$$= \frac{10^{-2}}{3^{-2}}$$

Properties of Exponents Example

$$\left(\frac{10}{3}\right)^{-2}$$

$$= \frac{10^{-2}}{3^{-2}}$$

$$= \frac{3^2}{10^2}$$

Properties of Exponents Example

$$\left(\frac{10}{3}\right)^{-2}$$

$$= \frac{10^{-2}}{3^{-2}}$$

$$= \frac{3^2}{10^2}$$

$$= \boxed{\frac{9}{100}}$$

Problem

Simplify the following expression:

$$\frac{8x^{-3}yz^5}{2x^{-5}y^4z^2}$$

$$\frac{8x^{-3}yz^{5}}{2x^{-5}y^{4}z^{2}}$$

$$\frac{8x^{-3}yz^5}{2x^{-5}y^4z^2}$$

$$= \frac{8x^{-3}yz^5z^{-2}}{2x^{-5}y^4}$$

$$\frac{8x^{-3}yz^5}{2x^{-5}y^4z^2}$$

$$= \frac{8x^{-3}yz^5z^{-2}}{2x^{-5}y^4}$$

$$= \frac{8x^{-3}yz^3}{2x^{-5}y^4}$$

$$\frac{8x^{-3}yz^5}{2x^{-5}y^4z^2}$$

$$= \frac{8x^{-3}yz^5z^{-2}}{2x^{-5}y^4}$$

$$= \frac{8x^{-3}yz^3}{2x^{-5}y^4}$$

$$= \frac{8x^{-3}x^5yz^3}{2y^4}$$

$$= \frac{8x^2yz^3}{2y^4}$$

$$= \frac{8x^2yz^3}{2y^4}$$

$$= \frac{8x^2z^3}{2y^{-1}y^4}$$

$$= \frac{8x^2yz^3}{2y^4}$$

$$= \frac{8x^2z^3}{2y^{-1}y^4}$$

$$= \frac{8x^2z^3}{2y^3}$$

$$= \frac{8x^2yz^3}{2y^4}$$

$$= \frac{8x^2z^3}{2y^{-1}y^4}$$

$$= \frac{8x^2z^3}{2y^3}$$

$$= \boxed{\frac{4x^2z^3}{y^3}}$$

More Problems with Exponents

Simplify the following expressions:

$$\left(5x^4y^{-2}\right)^{-2} \qquad \frac{3 * 6 - 2^3}{14 - 3^2}$$

$$\sqrt[2]{3^2 + 4^2} \qquad (-2)^4$$

$\sqrt[2]{\heartsuit}$ or $\heartsuit^{1/2}$ means to take the *square root* of $\heartsuit$, i.e. $\sqrt[2]{\heartsuit^2} = \heartsuit$.

Applications to Geometry

Important Formulas from Geometry

Applications to Geometry

$$\text{Area } \bigcirc = \pi r^2$$

$$\text{Circumference } \bigcirc = 2\pi r$$

$$\text{Area } \triangle = \frac{1}{2}bh$$

Note: r - radius of $\bigcirc$, b - base of $\triangle$, h - height of $\triangle$

Algebra Applied to Geometry

Problems

1. What is the circumference of a circle with radius $3\diamondsuit$ feet?

2. What is the area of a triangle with base equal to 1 foot and height equal to $6\heartsuit$ inches?

Algebra Applied to Geometry

Problems

1. What is the circumference of a circle with radius $3\diamondsuit$ feet?

   Solution: $2\pi(3\diamondsuit) = 6\pi\diamondsuit$ feet

2. What is the area of a triangle with base equal to 1 foot and height equal to $6\heartsuit$ inches?

   Solution: $\frac{1}{2}(1)\left(\frac{1}{2}\heartsuit\right) = \frac{1}{4}\heartsuit$ feet^2

Evaluating Algebraic Expressions for Given Values.

Evaluating Expressions for Given Values

Problems

Evaluate the expression $y = \sqrt[2]{2x + 1}$ for $x = 4$.

Evaluating Expressions with Square Roots for Given Values

$$y = \sqrt[2]{2x + 1}$$

Evaluating Expressions with Square Roots for Given Values

$$y = \sqrt[2]{2x + 1}$$

$$y = \sqrt[2]{2(4) + 1}$$

Evaluating Expressions with Square Roots for Given Values

$$y = \sqrt[2]{2x + 1}$$

$$y = \sqrt[2]{2(4) + 1}$$

$$y = \sqrt[2]{8 + 1}$$

Evaluating Expressions with Square Roots for Given Values

$$y = \sqrt[2]{2x + 1}$$

$$y = \sqrt[2]{2(4) + 1}$$

$$y = \sqrt[2]{8 + 1}$$

$$y = \sqrt[2]{9}$$

Evaluating Expressions with Square Roots for Given Values

$$y = \sqrt[2]{2x + 1}$$

$$y = \sqrt[2]{2(4) + 1}$$

$$y = \sqrt[2]{8 + 1}$$

$$y = \sqrt[2]{9}$$

$$\boxed{y = 3}$$

Evaluating Expressions with Cube Roots for Given Values

Problems

Evaluate the expression $\bigstar = \frac{1}{5}\sqrt[3]{\alpha}$, where

$\alpha = -125$.

$\sqrt[3]{\heartsuit}$ or $\heartsuit^{1/3}$ means to take the *cube root* of $\heartsuit$, i.e. $\sqrt[3]{\heartsuit^3} = \heartsuit$.

Evaluating Expressions with Cube Roots for Given Values

$$\bigstar = \frac{1}{5}\sqrt[3]{\alpha}$$

Evaluating Expressions with Cube Roots for Given Values

$$\bigstar = \frac{1}{5}\sqrt[3]{\alpha}$$

$$\bigstar = \frac{1}{5}\sqrt[3]{-125}$$

Evaluating Expressions with Cube Roots for Given Values

$$\bigstar = \frac{1}{5}\sqrt[3]{\alpha}$$

$$\bigstar = \frac{1}{5}\sqrt[3]{-125}$$

$$\bigstar = \frac{1}{5}(-5)$$

Evaluating Expressions with Cube Roots for Given Values

$$\bigstar = \frac{1}{5}\sqrt[3]{\alpha}$$

$$\bigstar = \frac{1}{5}\sqrt[3]{-125}$$

$$\bigstar = \frac{1}{5}(-5)$$

$$\boxed{\bigstar = -1}$$

Evaluating Expressions with Cube Roots for Given Values

$$\bigstar = \frac{1}{5}\sqrt[3]{\alpha}$$

$$\bigstar = \frac{1}{5}\sqrt[3]{-125}$$

$$\bigstar = \frac{1}{5}(-5)$$

$$\boxed{\bigstar = -1}$$

Note: $(negative\#)^{odd\#}$ is negative and $(negative\#)^{even\#}$ is positive

Evaluating Absolute Values || for Given Values

Problems

Evaluate the expression $\spadesuit = |\clubsuit - 5|$, where $\clubsuit = 2$.

Evaluating Absolute Values || for Given Values

Problems

Evaluate the expression $\spadesuit = |\clubsuit - 5|$, where $\clubsuit = 2$.

$$\spadesuit = |\clubsuit - 5|$$

Evaluating Absolute Values || for Given Values

Problems

Evaluate the expression $\spadesuit = |\clubsuit - 5|$, where $\clubsuit = 2$.

$$\spadesuit = |\clubsuit - 5|$$

$$\spadesuit = |2 - 5|$$

Evaluating Absolute Values || for Given Values

Problems

Evaluate the expression $\spadesuit = |\clubsuit - 5|$, where $\clubsuit = 2$.

$$\spadesuit = |\clubsuit - 5|$$

$$\spadesuit = |2 - 5|$$

$$\spadesuit = |-3|$$

Evaluating Absolute Values || for Given Values

Problems

Evaluate the expression $\spadesuit = |\clubsuit - 5|$, where $\clubsuit = 2$.

$$\spadesuit = |\clubsuit - 5|$$

$$\spadesuit = |2 - 5|$$

$$\spadesuit = |-3|$$

$$\boxed{\spadesuit = 3}$$

Note: $|-\spadesuit| = \spadesuit$ and $|\spadesuit| = \spadesuit$, where $\spadesuit \geq 0$ is positive.

Working with Data.

Which Algebraic Expression Represents the Data?

x	y
1	2
2	6
3	10
4	14

Which Algebraic Expression Represents the Data?

x	y
1	2
2	6
3	10
4	14

$y = 3x + 1$

$y = x + 4$

$y = 4x - 2$

Which Algebraic Expression Represents the Data?

x	y
1	2
2	6
3	10
4	14

$y = 3x + 1$

$y = x + 4$

$\boxed{y = 4x - 2}$

What Algebraic Expression Represents the Data?

θ	Δ
5	7
7	9
0	2
-3	-1
-8	-6

What Algebraic Expression Represents the Data?

θ	Δ
5	7
7	9
0	2
-3	-1
-8	-6

The following expression models the data.

$$\Delta = \theta - \alpha$$

Find α.

What Algebraic Expression Represents the Data?

θ	Δ
5	7
7	9
0	2
-3	-1
-8	-6

The following expression models the data.

$$\Delta = \theta - \alpha$$

Find α.

$\boxed{\alpha = -2}$

What was the Original Data Set?

$\diamondsuit$	$\heartsuit$
-1	
0	
1	
8	
125	

$$\heartsuit = \sqrt[3]{\diamondsuit} - 5$$

What was the Original Data Set?

$\diamondsuit$	$\heartsuit$
-1	-6
0	-5
1	-4
8	-3
125	0

$$\heartsuit = \sqrt[3]{\diamondsuit} - 5$$

Next Time...

Relations, Domains & Ranges

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