Algebra Lecture 21

Crista Moreno

December 7, 2016

Topics for Today

Topics

- Sum and Difference of Radicals
- Solving Radical Expressions

Recall the definition of

Exponentiation.

Exponentiation

$$a^n = \overbrace{a * a * \cdots * a}^{n}$$

Properties of Exponents

$$a^0 \equiv 1 \qquad a^{\frac{1}{n}} \equiv \sqrt[n]{a}$$

$$a^n a^m = a^{n+m} \qquad a^n a^{-m} = a^{n+-m}$$

$$a^{-1} \equiv \frac{1}{a} \qquad a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad \left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$$

Properties of Exponents

Properties of a^n

$a^n a^m = a^{n+m}$

Properties of Exponents

Properties of a^n

$a^n a^m = a^{n+m}$

Why?

Properties of Exponents

Properties of a^n

$$a^n a^m = a^{n+m}$$

Why?

$$a^n * a^m = \overbrace{a * a * \cdots * a}^{n} * \overbrace{a * a * \cdots * a}^{m} = a^{n+m}$$

Properties of Exponents

Properties of a^n

$$a^n a^m = a^{n+m}$$

Why?

$$a^n * a^m = \overbrace{a * a * \cdots * a}^{n} * \overbrace{a * a * \cdots * a}^{m} = a^{n+m}$$

$$3^2 3^3 = \overbrace{3 * 3}^{2} * \overbrace{3 * 3 * 3}^{3}$$

$$= \overbrace{3 * 3 * 3 * 3 * 3}^{5} = 3^5 = \boxed{3^{2+3}}$$

Properties of Exponents

Properties of a^n

$$(a^n)^m = a^{n*m}$$

Properties of Exponents

Properties of a^n

$(a^n)^m = a^{n*m}$

Why?

Properties of Exponents

Properties of a^n

$$(a^n)^m = a^{n*m}$$

Why?

$$(a^n)^m = \overbrace{a^n * a^n * \cdots * a^n}^{m}$$

$$= \overbrace{\underbrace{a * \cdots * a}_{n} * \cdots * \underbrace{a * \cdots * a}_{n}}^{m} = a^{n*m}$$

Properties of Exponents

Properties of a^n

$$(a^n)^m = a^{n*m}$$

Why?

$$(a^n)^m = \overbrace{a^n * a^n * \cdots * a^n}^{m}$$

$$= \overbrace{\underbrace{a * \cdots * a}_{n} * \cdots * \underbrace{a * \cdots * a}_{n}}^{m} = a^{n*m}$$

$$(4^2)^3 = \overbrace{4^2 * 4^2 * 4^2}^{3} = \overbrace{4 * 4 * 4 * 4 * 4 * 4}^{6} = \boxed{4^{2*3}}$$

Properties of Exponents

Properties of $\left(\frac{a}{b}\right)^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Properties of Exponents

Properties of $\left(\frac{a}{b}\right)^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Why?

Properties of Exponents

Properties of $\left(\frac{a}{b}\right)^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Why?

$$\left(\frac{a}{b}\right)^n = \overbrace{\left(\frac{a}{b}\right) * \cdots * \left(\frac{a}{b}\right)}^{n} = \frac{\overbrace{a * \cdots * a}^{n}}{\underbrace{b * \cdots * b}_{n}} = \frac{a^n}{b^n}$$

Properties of Exponents

Properties of $\left(\frac{a}{b}\right)^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Why?

$$\left(\frac{a}{b}\right)^n = \overbrace{\left(\frac{a}{b}\right) * \cdots * \left(\frac{a}{b}\right)}^{n} = \frac{\overbrace{a * \cdots * a}^{n}}{\underbrace{b * \cdots * b}_{n}} = \frac{a^n}{b^n}$$

$$\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right) * \left(\frac{3}{4}\right) = \frac{3 * 3}{4 * 4} = \boxed{\frac{3^2}{4^2}}$$

Properties of Exponents

Definition a^{-n}

$$a^{-n} \equiv \frac{1}{a^n} \qquad \& \qquad a^m a^{-n} = a^{m+(-n)}$$

Properties of Exponents

Definition a^{-n}

$$a^{-n} \equiv \frac{1}{a^n} \qquad \& \qquad a^m a^{-n} = a^{m+(-n)}$$

Why?

Properties of Exponents

Definition a^{-n}

$$a^{-n} \equiv \frac{1}{a^n} \qquad \& \qquad a^m a^{-n} = a^{m+(-n)}$$

Why?

$$a^m * a^{-n} = \overbrace{a * a * \cdots * a}^{m} * \frac{1}{\underbrace{a * a * \cdots * a}_{n}}$$

Properties of Exponents

Definition a^{-n}

$$a^{-n} \equiv \frac{1}{a^n} \qquad \& \qquad a^m a^{-n} = a^{m+(-n)}$$

Why?

$$a^m * a^{-n} = \overbrace{a * a * \cdots * a}^{m} * \frac{1}{\underbrace{a * a * \cdots * a}_{n}}$$

$$3^2 3^{-4} = \overbrace{\cancel{3} * \cancel{3}}^{2} * \frac{1}{\underbrace{\cancel{3} * \cancel{3} * 3 * 3}_{4}} = \frac{1}{3 * 3} = \frac{1}{3^2} = \boxed{3^{-2}}$$

Properties of Exponents

Definition $a^{1/n}$

$$a^{1/n} \equiv \sqrt[n]{a} \qquad \& \qquad \left(a^n\right)^{1/n} = a^1 = a$$

Definition $a^{1/n}$

$a^{1/n} \equiv \sqrt[n]{a}$ & $(a^n)^{1/n} = a^1 = a$

Why?

Definition $a^{1/n}$

$$a^{1/n} \equiv \sqrt[n]{a} \qquad \& \qquad \left(a^n\right)^{1/n} = a^1 = a$$

Why?

$$\left(a^n\right)^{1/n} = \sqrt[n]{a^n} = a^1 = a$$

Definition $a^{1/n}$

$$a^{1/n} \equiv \sqrt[n]{a} \qquad \& \qquad \left(a^n\right)^{1/n} = a^1 = a$$

Why?

$$\left(a^n\right)^{1/n} = \sqrt[n]{a^n} = a^1 = a$$

$$\left(3^4\right)^{1/4} = \sqrt[4]{3^4} = \boxed{3}$$

Properties of Exponents

Even Root n $\sqrt[n]{a}$

- a positive → result is positive.
- a negative → NOT POSSIBLE!

Odd Root n $\sqrt[n]{a}$

- a positive → result is positive.
- a negative → result is negative.

Arithmetic examples of simplifying radical expressions.

Simplify the following expression

$$7\sqrt[2]{3} + 8\sqrt[2]{3} - \sqrt[2]{3}$$

Simplify the following expression

$$7\sqrt[2]{3} + 8\sqrt[2]{3} - \sqrt[2]{3}$$

$$= 7\sqrt[2]{3} + 8\sqrt[2]{3} - 1\sqrt[2]{3}$$

Simplify the following expression

$$7\sqrt[2]{3} + 8\sqrt[2]{3} - \sqrt[2]{3}$$

$$= 7\sqrt[2]{3} + 8\sqrt[2]{3} - 1\sqrt[2]{3}$$

$$= (7 + 8 - 1)\sqrt[2]{3}$$

Simplify the following expression

$$7\sqrt[2]{3} + 8\sqrt[2]{3} - \sqrt[2]{3}$$

$$= 7\sqrt[2]{3} + 8\sqrt[2]{3} - 1\sqrt[2]{3}$$

$$= (7 + 8 - 1)\sqrt[2]{3}$$

$$= \boxed{14\sqrt[2]{3}}$$

Simplify the following expression

$$8\sqrt[3]{11} + 15\sqrt[3]{11}$$

Simplify the following expression

$$8\sqrt[3]{11} + 15\sqrt[3]{11}$$

$$= (8 + 15)\sqrt[3]{11}$$

Simplify the following expression

$$8\sqrt[3]{11} + 15\sqrt[3]{11}$$

$$= (8 + 15)\sqrt[3]{11}$$

$$= \boxed{23\sqrt[3]{11}}$$

Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9 * 7} + 9\sqrt[2]{7}$$

Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9 * 7} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9}\sqrt[2]{7} + 9\sqrt[2]{7}$$

Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9 * 7} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9}\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 2 * 3\sqrt[2]{7} + 9\sqrt[2]{7}$$

Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9 * 7} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9}\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 2 * 3\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 6\sqrt[2]{7} + 9\sqrt[2]{7}$$

Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9 * 7} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9}\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 2 * 3\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 6\sqrt[2]{7} + 9\sqrt[2]{7} = (6 + 9)\sqrt[2]{7}$$

Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9 * 7} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9}\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 2 * 3\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 6\sqrt[2]{7} + 9\sqrt[2]{7} = (6 + 9)\sqrt[2]{7} = \boxed{15\sqrt[2]{7}}$$

Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9 * 3} - 2\sqrt[2]{3}$$

Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9 * 3} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9}\sqrt[2]{3} - 2\sqrt[2]{3}$$

Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9 * 3} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9}\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 2 * 3\sqrt[2]{3} - 2\sqrt[2]{3}$$

Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9 * 3} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9}\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 2 * 3\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 6\sqrt[2]{3} - 2\sqrt[2]{3}$$

Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9 * 3} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9}\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 2 * 3\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 6\sqrt[2]{3} - 2\sqrt[2]{3} = (6 - 2)\sqrt[2]{3}$$

Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9 * 3} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9}\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 2 * 3\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 6\sqrt[2]{3} - 2\sqrt[2]{3} = (6 - 2)\sqrt[2]{3} = \boxed{4\sqrt[2]{3}}$$

Algebraic examples of simplifying radical expressions.

Simplify the following expression

$$\sqrt[2]{20x + 20} + \sqrt[2]{5x + 5}$$

Simplify the following expression

$$\sqrt[2]{20x+20}+\sqrt[2]{5x+5}$$

$$=\sqrt[2]{4(5x+5)}+\sqrt[2]{5x+5}$$

Simplify the following expression

$$\sqrt[2]{20x + 20} + \sqrt[2]{5x + 5}$$

$$= \sqrt[2]{4(5x + 5)} + \sqrt[2]{5x + 5}$$

$$= \sqrt[2]{4}\sqrt[2]{(5x + 5)} + \sqrt[2]{5x + 5}$$

Simplify the following expression

$$\sqrt[2]{20x + 20} + \sqrt[2]{5x + 5}$$

$$= \sqrt[2]{4(5x + 5)} + \sqrt[2]{5x + 5}$$

$$= \sqrt[2]{4}\sqrt[2]{(5x + 5)} + \sqrt[2]{5x + 5}$$

$$= 2\sqrt[2]{(5x + 5)} + \sqrt[2]{5x + 5}$$

Simplify the following expression

$$\sqrt[2]{20x + 20} + \sqrt[2]{5x + 5}$$

$$= \sqrt[2]{4(5x + 5)} + \sqrt[2]{5x + 5}$$

$$= \sqrt[2]{4}\sqrt[2]{(5x + 5)} + \sqrt[2]{5x + 5}$$

$$= 2\sqrt[2]{(5x + 5)} + \sqrt[2]{5x + 5}$$

$$= (2 + 1)\sqrt[2]{(5x + 5)}$$

Simplify the following expression

$$\sqrt[2]{20x+20} + \sqrt[2]{5x+5}$$

$$= \sqrt[2]{4(5x+5)} + \sqrt[2]{5x+5}$$

$$= \sqrt[2]{4}\sqrt[2]{(5x+5)} + \sqrt[2]{5x+5}$$

$$= 2\sqrt[2]{(5x+5)} + \sqrt[2]{5x+5}$$

$$= (2+1)\sqrt[2]{(5x+5)} = \boxed{3\sqrt[2]{(5x+5)}}$$

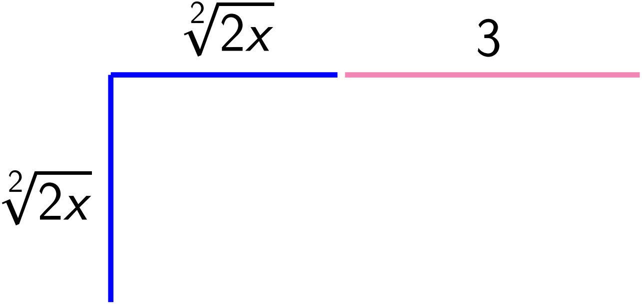
Simplify the following expression

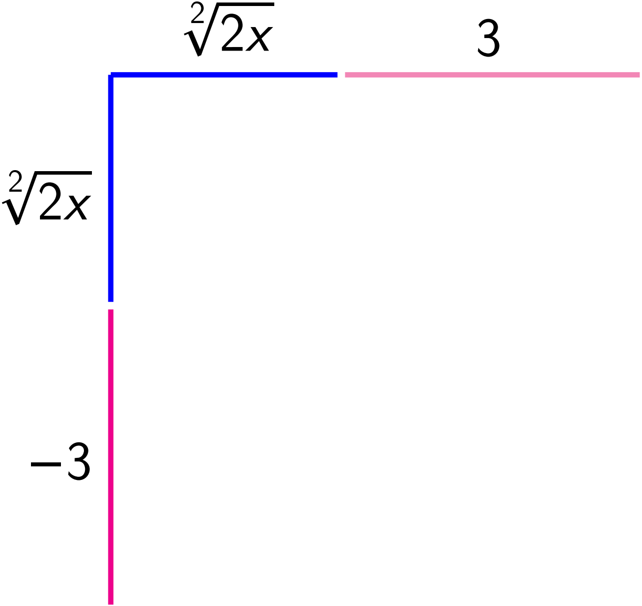
$$(\sqrt[2]{2x}+3)(\sqrt[2]{2x}-3)$$

Recall how to multiply such quantities.

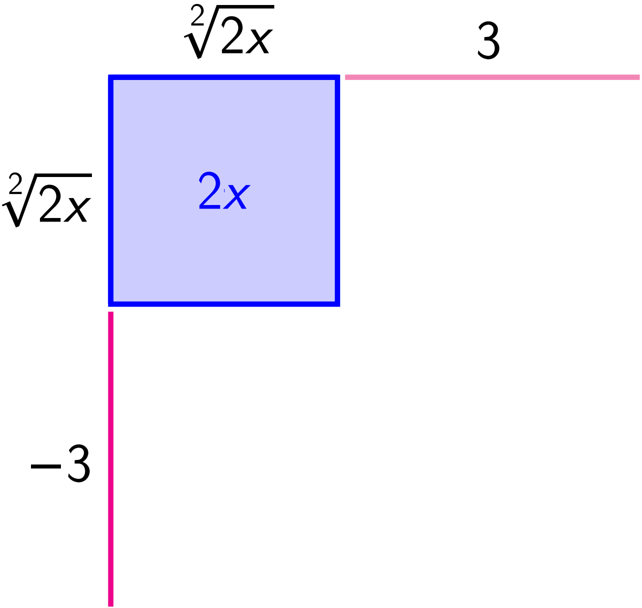
$$\sqrt[2]{2x}$$

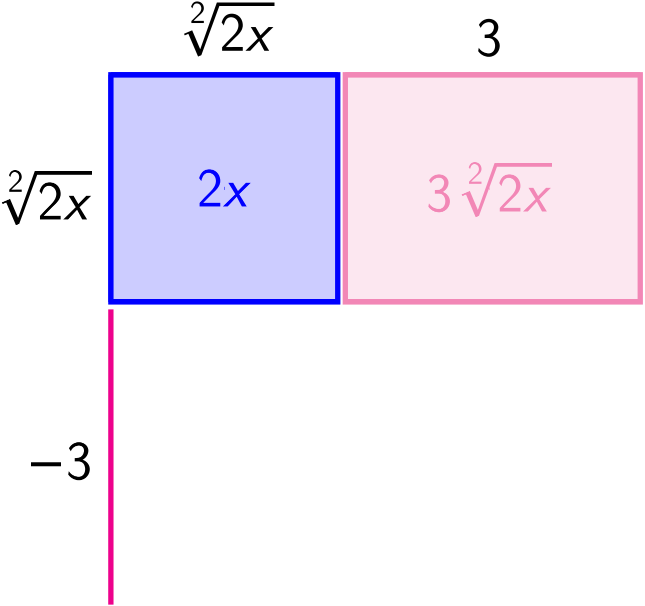
$\sqrt[2]{2x}$ 3


$\sqrt[2]{2x}$
3
$\sqrt[2]{2x}$

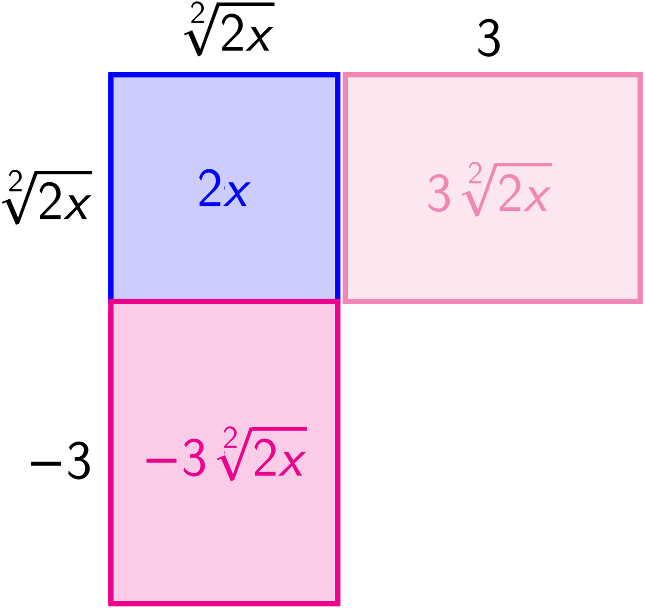


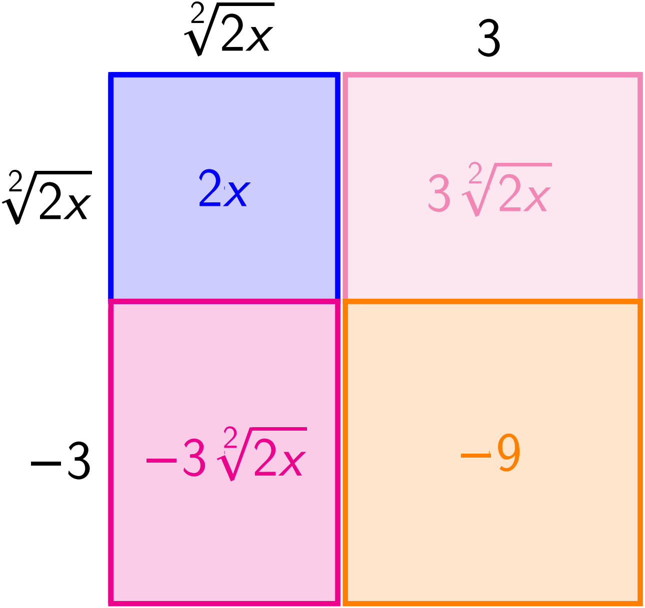
$\sqrt[2]{2x}$
3
$\sqrt[2]{2x}$
-3

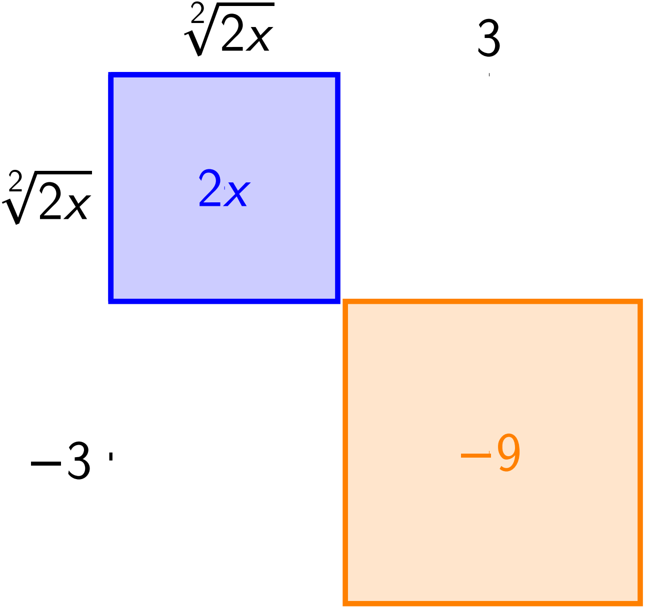

$\sqrt[2]{2x}$
3
$\sqrt[2]{2x}$
$2x$
-3



$\sqrt[2]{2x}$
3
$\sqrt[2]{2x}$
$2x$
$3\sqrt[2]{2x}$
-3


$\sqrt[2]{2x}$
3
$\sqrt[2]{2x}$
$2x$
$3\sqrt[2]{2x}$
-3
$-3\sqrt[2]{2x}$


$\sqrt[2]{2x}$
3
$\sqrt[2]{2x}$
$2x$
$3\sqrt[2]{2x}$
-3
$-3\sqrt[2]{2x}$
-9


$\sqrt[2]{2x}$
3
$\sqrt[2]{2x}$
$2x$
-3
-9

Simplify the following expression

$$(\sqrt[2]{2x} + 3)(\sqrt[2]{2x} - 3)$$

Simplify the following expression

$$(\sqrt[2]{2x} + 3)(\sqrt[2]{2x} - 3)$$

$$= \sqrt[2]{2x}\sqrt[2]{2x} + 3\sqrt[2]{2x} - 3\sqrt[2]{2x} - 3 * 3$$

Simplify the following expression

$$(\sqrt[2]{2x}+3)(\sqrt[2]{2x}-3)$$

$$=\sqrt[2]{2x}\sqrt[2]{2x}+3\sqrt[2]{2x}-3\sqrt[2]{2x}-3*3$$

$$=2x+\cancel{3\sqrt[2]{2x}}\cancel{-3\sqrt[2]{2x}}-9$$

Simplify the following expression

$$(\sqrt[2]{2x}+3)(\sqrt[2]{2x}-3)$$

$$= \sqrt[2]{2x}\sqrt[2]{2x}+3\sqrt[2]{2x}-3\sqrt[2]{2x}-3*3$$

$$= 2x + \cancel{3\sqrt[2]{2x}} \cancel{-3\sqrt[2]{2x}} - 9$$

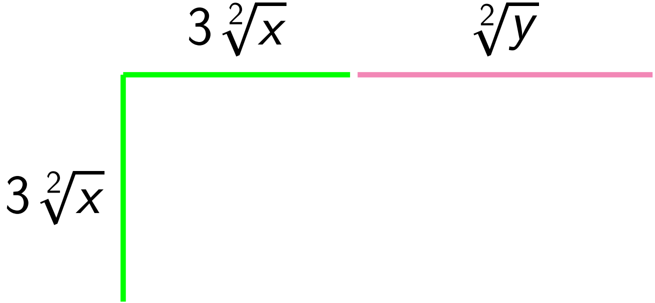
$$= \boxed{2x-9}$$

Simplify the following expression

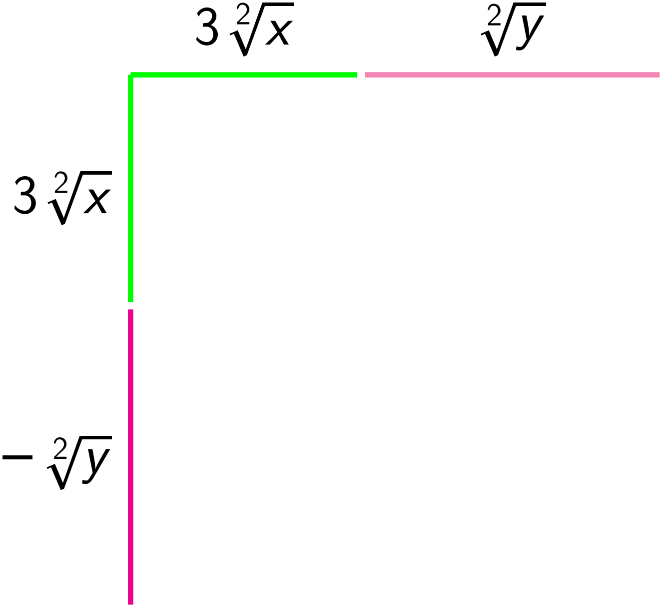
$$\left(3\sqrt[2]{x} + \sqrt[2]{y}\right)\left(3\sqrt[2]{x} - \sqrt[2]{y}\right)$$

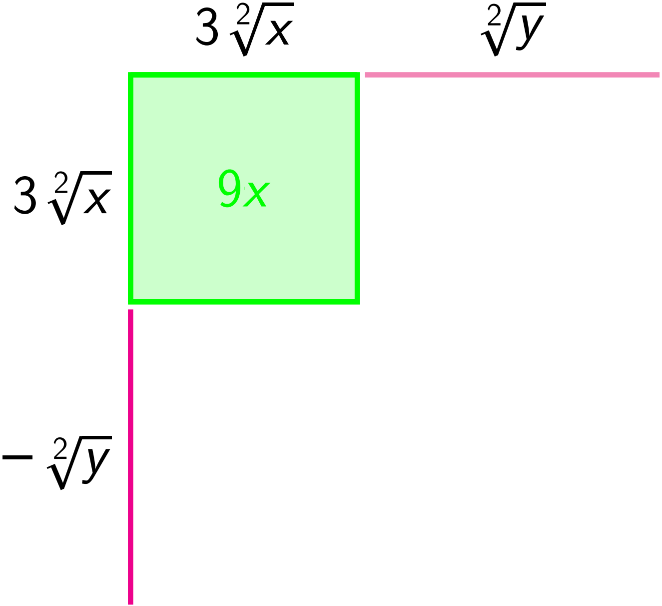
$$3\sqrt[2]{x}$$

$3\sqrt[2]{x}$ $\sqrt[2]{y}$

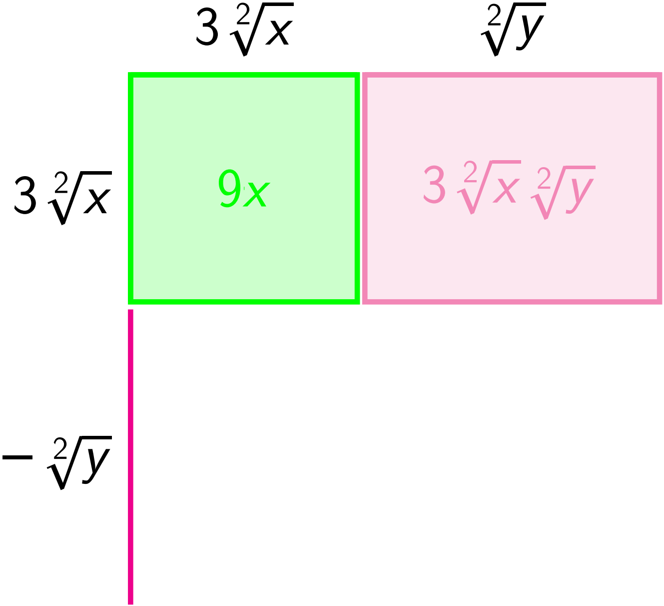


$3\sqrt[2]{x}$
$\sqrt[2]{y}$
$3\sqrt[2]{x}$


$3\sqrt[2]{x}$
$\sqrt[2]{y}$
$3\sqrt[2]{x}$
$-\sqrt[2]{y}$

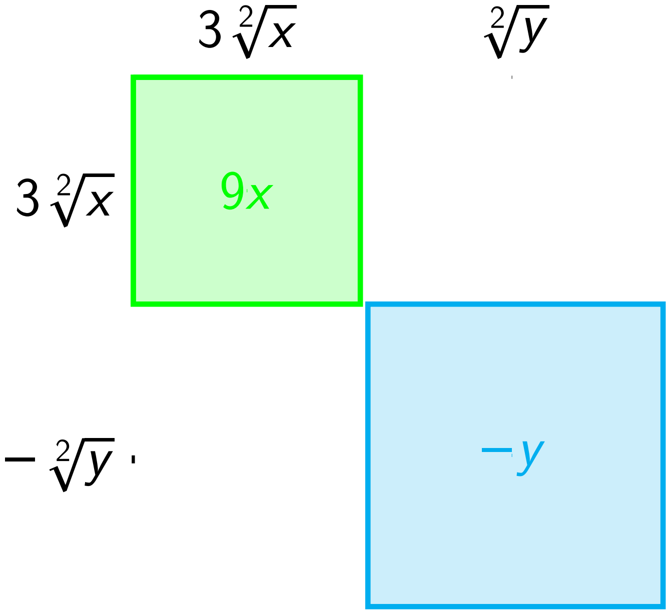


$3\sqrt[2]{x}$
$\sqrt[2]{y}$
$3\sqrt[2]{x}$
$9x$
$-\sqrt[2]{y}$


$3\sqrt[2]{x}$
$\sqrt[2]{y}$
$3\sqrt[2]{x}$
$9x$
$3\sqrt[2]{x}\sqrt[2]{y}$
$-\sqrt[2]{y}$

	$3\sqrt[2]{x}$	$\sqrt[2]{y}$
$3\sqrt[2]{x}$	$9x$	$3\sqrt[2]{x}\sqrt[2]{y}$
$-\sqrt[2]{y}$	$-3\sqrt[2]{x}\sqrt[2]{y}$	

	$3\sqrt[2]{x}$	$\sqrt[2]{y}$
$3\sqrt[2]{x}$	$9x$	$3\sqrt[2]{x}\sqrt[2]{y}$
$-\sqrt[2]{y}$	$-3\sqrt[2]{x}\sqrt[2]{y}$	$-y$



$3\sqrt[2]{x}$
$\sqrt[2]{y}$
$3\sqrt[2]{x}$
$9x$
$-\sqrt[2]{y}$
$-y$

Simplify the following expression

$$\left(3\sqrt[2]{x}+\sqrt[2]{y}\right)\left(3\sqrt[2]{x}-\sqrt[2]{y}\right)$$

Simplify the following expression

$$(3\sqrt[2]{x} + \sqrt[2]{y})(3\sqrt[2]{x} - \sqrt[2]{y})$$

$$= 3\sqrt[2]{x} * 3\sqrt[2]{x} +$$

$$\sqrt[2]{y} * 3\sqrt[2]{x} - 3\sqrt[2]{x} * \sqrt[2]{y} - \sqrt[2]{y} * \sqrt[2]{y}$$

Simplify the following expression

$$(3\sqrt[2]{x} + \sqrt[2]{y})(3\sqrt[2]{x} - \sqrt[2]{y})$$

$$= 3\sqrt[2]{x} * 3\sqrt[2]{x} +$$

$$\sqrt[2]{y} * 3\sqrt[2]{x} - 3\sqrt[2]{x} * \sqrt[2]{y} - \sqrt[2]{y} * \sqrt[2]{y}$$

$$= 9x + \cancel{\sqrt[2]{y} * 3\sqrt[2]{x}} \cancel{- 3\sqrt[2]{x} * \sqrt[2]{y}} - y$$

Simplify the following expression

$$(3\sqrt[2]{x} + \sqrt[2]{y})(3\sqrt[2]{x} - \sqrt[2]{y})$$

$$= 3\sqrt[2]{x} * 3\sqrt[2]{x} +$$

$$\sqrt[2]{y} * 3\sqrt[2]{x} - 3\sqrt[2]{x} * \sqrt[2]{y} - \sqrt[2]{y} * \sqrt[2]{y}$$

$$= 9x + \cancel{\sqrt[2]{y} * 3\sqrt[2]{x}} \cancel{- 3\sqrt[2]{x} * \sqrt[2]{y}} - y$$

$$= \boxed{9x - y}$$

Rationalizing a fraction with a radical expression in the denominator.

Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}}$$

Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}} \frac{\sqrt[2]{7}}{\sqrt[2]{7}}$$

Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}}\frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}}$$

Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}}\frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}} = \boxed{\frac{2\sqrt[2]{7}}{7}}$$

Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}}\frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}} = \boxed{\frac{2\sqrt[2]{7}}{7}}$$

$$\frac{8}{\sqrt[2]{2}}$$

Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}}\frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}} = \boxed{\frac{2\sqrt[2]{7}}{7}}$$

$$\frac{8}{\sqrt[2]{2}} = \frac{8}{\sqrt[2]{2}}\frac{\sqrt[2]{2}}{\sqrt[2]{2}}$$

Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}}\frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}} = \boxed{\frac{2\sqrt[2]{7}}{7}}$$

$$\frac{8}{\sqrt[2]{2}} = \frac{8}{\sqrt[2]{2}}\frac{\sqrt[2]{2}}{\sqrt[2]{2}} = \frac{8\sqrt[2]{2}}{\sqrt[2]{2}\sqrt[2]{2}}$$

Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}}\frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}} = \boxed{\frac{2\sqrt[2]{7}}{7}}$$

$$\frac{8}{\sqrt[2]{2}} = \frac{8}{\sqrt[2]{2}}\frac{\sqrt[2]{2}}{\sqrt[2]{2}} = \frac{8\sqrt[2]{2}}{\sqrt[2]{2}\sqrt[2]{2}} = \frac{8\sqrt[2]{2}}{2}$$

Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}}\frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}} = \boxed{\frac{2\sqrt[2]{7}}{7}}$$

$$\frac{8}{\sqrt[2]{2}} = \frac{8}{\sqrt[2]{2}}\frac{\sqrt[2]{2}}{\sqrt[2]{2}} = \frac{8\sqrt[2]{2}}{\sqrt[2]{2}\sqrt[2]{2}} = \frac{8\sqrt[2]{2}}{2}$$

$$= \boxed{4\sqrt[2]{2}}$$

Solving radical equations.

Solve for the unknown variable ♥

$$\sqrt[2]{\heartsuit} = 4$$

Solve for the unknown variable ♥

$$\sqrt[2]{\heartsuit} = 4$$

$$\left(\sqrt[2]{\heartsuit}\right)^{2} = 4^{2}$$

Solve for the unknown variable ♥

$$\sqrt[2]{\heartsuit} = 4$$

$$\left(\sqrt[2]{\heartsuit}\right)^{2} = 4^{2}$$

$$\left(\sqrt[\not{2}]{\heartsuit}\right)^{\not{2}} = 16$$

Solve for the unknown variable ♥

$$\sqrt[2]{\heartsuit} = 4$$

$$\left(\sqrt[2]{\heartsuit}\right)^2 = 4^2$$

$$\left(\sqrt[\not{2}]{\heartsuit}\right)^{\not{2}} = 16$$

$$\boxed{\heartsuit = 16}$$

Solve for the unknown variable ♦

$$\sqrt[2]{3\blacklozenge + 7} = 5$$

Solve for the unknown variable ♦

$$\sqrt[2]{3\blacklozenge + 7} = 5$$

$$\left(\sqrt[2]{3\blacklozenge + 7}\right)^{2} = 5^{2}$$

Solve for the unknown variable ♦

$$\sqrt[2]{3\blacklozenge + 7} = 5$$

$$\left(\sqrt[2]{3\blacklozenge + 7}\right)^{2} = 5^{2}$$

$$3\blacklozenge + 7 = 25$$

Solve for the unknown variable ♦

$$\sqrt[2]{3\blacklozenge + 7} = 5$$

$$\left(\sqrt[2]{3\blacklozenge + 7}\right)^{2} = 5^{2}$$

$$3\blacklozenge + 7 = 25$$

$$3\blacklozenge = 18$$

Solve for the unknown variable ♦

$$\sqrt[2]{3\blacklozenge + 7} = 5$$

$$\left(\sqrt[2]{3\blacklozenge + 7}\right)^2 = 5^2$$

$$3\blacklozenge + 7 = 25$$

$$3\blacklozenge = 18$$

$$\boxed{\blacklozenge = 6}$$

Solve for the unknown variable ♣

$$\sqrt[3]{\clubsuit} = -4$$

Solve for the unknown variable ♣

$$\sqrt[3]{\clubsuit} = -4$$

$$\left(\sqrt[3]{\clubsuit}\right)^{3} = -4^{3}$$

Solve for the unknown variable ♣

$$\sqrt[3]{\clubsuit} = -4$$

$$\left(\sqrt[3]{\clubsuit}\right)^3 = -4^3$$

$$\left(\sqrt[\cancel{3}]{\clubsuit}\right)^{\cancel{3}} = -4^3$$

Solve for the unknown variable ♣

$$\sqrt[3]{♣} = -4$$

$$\left(\sqrt[3]{♣}\right)^{3} = -4^{3}$$

$$\left(\sqrt[\not{3}]{♣}\right)^{\not{3}} = -4^{3}$$

$$\boxed{♣ = -64}$$

Solve for the unknown variable ♡

$$\heartsuit = \sqrt[2]{\heartsuit + 6}$$

Solve for the unknown variable ♡

$$\heartsuit = \sqrt[2]{\heartsuit + 6}$$

$$\heartsuit^{2} = \left(\sqrt[2]{\heartsuit + 6}\right)^{2}$$

Solve for the unknown variable ♡

$$\heartsuit = \sqrt[2]{\heartsuit + 6}$$

$$\heartsuit^2 = \left(\sqrt[2]{\heartsuit + 6}\right)^2$$

$$\heartsuit^2 = \left(\sqrt[\cancel{2}]{\heartsuit + 6}\right)^{\cancel{2}}$$

Solve for the unknown variable ♡

$$\heartsuit^2 = \heartsuit + 6$$

Solve for the unknown variable ♡

$$\heartsuit^2 = \heartsuit + 6$$

$$\heartsuit^2 - \heartsuit - 6 = 0$$

Solve for the unknown variable $\heartsuit$

$$\heartsuit^2 = \heartsuit + 6$$

$$\heartsuit^2 - \heartsuit - 6 = 0$$

$$(\heartsuit - 3)(\heartsuit + 2) = 0$$

Solve for the unknown variable $\heartsuit$

$$\heartsuit^2 = \heartsuit + 6$$

$$\heartsuit^2 - \heartsuit - 6 = 0$$

$$(\heartsuit - 3)(\heartsuit + 2) = 0$$

$$\boxed{\heartsuit = -3}$$

Next Time

Applications of Radical Expressions