

Copyright 2016 Crista Moreno. Algebra Lecture 21 is made available under the Creative Commons Attribution-ShareAlike 4.0 International License.

To view a copy of this license, visit

<http://creativecommons.org/licenses/by-sa/4.0/>.



# Algebra Lecture 21

Crista Moreno

December 7, 2016

# Topics for Today

## Topics

- Sum and Difference of Radicals
- Solving Radical Expressions

Recall the definition of  
Exponentiation.

# Exponentiation

$$a^n = \overbrace{a * a * \cdots * a}^n$$

## Properties of Exponents

$$a^0 \equiv 1$$

$$a^n a^m = a^{n+m}$$

$$a^{-1} \equiv \frac{1}{a}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{\frac{1}{n}} \equiv \sqrt[n]{a}$$

$$a^n a^{-m} = a^{n+-m}$$

$$a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$$

# Properties of Exponents

## Properties of $a^n$

$$a^n a^m = a^{n+m}$$

# Properties of Exponents

## Properties of $a^n$

$$a^n a^m = a^{n+m}$$

Why?



# Properties of Exponents

## Properties of $a^n$

$$a^n a^m = a^{n+m}$$

Why?

$$a^n * a^m = \overbrace{a * a * \cdots * a}^n * \overbrace{a * a * \cdots * a}^m = a^{n+m}$$

# Properties of Exponents

## Properties of $a^n$

$$a^n a^m = a^{n+m}$$

Why?

$$a^n * a^m = \overbrace{a * a * \cdots * a}^n * \overbrace{a * a * \cdots * a}^m = a^{n+m}$$

$$3^2 3^3 = \overbrace{3 * 3}^2 * \overbrace{3 * 3 * 3}^3$$

$$= \overbrace{3 * 3 * 3 * 3 * 3}^5 = 3^5 = \boxed{3^{2+3}}$$

# Properties of Exponents

Properties of  $a^n$

$$(a^n)^m = a^{n*m}$$

# Properties of Exponents

## Properties of $a^n$

$$(a^n)^m = a^{n*m}$$

Why?

# Properties of Exponents

## Properties of $a^n$

$$(a^n)^m = a^{n*m}$$

Why?

$$\begin{aligned}(a^n)^m &= \overbrace{a^n * a^n * \dots * a^n}^m \\ &= \underbrace{a * \dots * a}_n * \dots * \underbrace{a * \dots * a}_n = a^{n*m}\end{aligned}$$

# Properties of Exponents

## Properties of $a^n$

$$(a^n)^m = a^{n*m}$$

Why?

$$\begin{aligned}(a^n)^m &= \overbrace{a^n * a^n * \dots * a^n}^m \\ &= \underbrace{a * \dots * a}_n * \dots * \underbrace{a * \dots * a}_n = a^{n*m}\end{aligned}$$

$$(4^2)^3 = \overbrace{4^2 * 4^2 * 4^2}^3 = \overbrace{4 * 4 * 4 * 4 * 4 * 4}^6 = \boxed{4^{2*3}}$$

# Properties of Exponents

Properties of  $\left(\frac{a}{b}\right)^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

# Properties of Exponents

Properties of  $\left(\frac{a}{b}\right)^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Why?



# Properties of Exponents

## Properties of $\left(\frac{a}{b}\right)^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Why?

$$\left(\frac{a}{b}\right)^n = \overbrace{\left(\frac{a}{b}\right) * \cdots * \left(\frac{a}{b}\right)}^n = \frac{\overbrace{a * \cdots * a}^n}{\underbrace{b * \cdots * b}_n} = \frac{a^n}{b^n}$$

# Properties of Exponents

## Properties of $\left(\frac{a}{b}\right)^n$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Why?

$$\left(\frac{a}{b}\right)^n = \overbrace{\left(\frac{a}{b}\right) * \cdots * \left(\frac{a}{b}\right)}^n = \frac{\overbrace{a * \cdots * a}^n}{\underbrace{b * \cdots * b}_n} = \frac{a^n}{b^n}$$

$$\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right) * \left(\frac{3}{4}\right) = \frac{3 * 3}{4 * 4} = \boxed{\frac{3^2}{4^2}}$$

# Properties of Exponents

Definition  $a^{-n}$

$$a^{-n} \equiv \frac{1}{a^n} \quad \& \quad a^m a^{-n} = a^{m+(-n)}$$

# Properties of Exponents

Definition  $a^{-n}$

$$a^{-n} \equiv \frac{1}{a^n} \quad \& \quad a^m a^{-n} = a^{m+(-n)}$$

Why?

# Properties of Exponents

## Definition $a^{-n}$

$$a^{-n} \equiv \frac{1}{a^n} \quad \& \quad a^m a^{-n} = a^{m+(-n)}$$

Why?

$$a^m * a^{-n} = \overbrace{a * a * \cdots * a}^m * \frac{1}{\underbrace{a * a * \cdots * a}_n}$$

# Properties of Exponents

## Definition $a^{-n}$

$$a^{-n} \equiv \frac{1}{a^n} \quad \& \quad a^m a^{-n} = a^{m+(-n)}$$

Why?

$$a^m * a^{-n} = \overbrace{a * a * \cdots * a}^m * \underbrace{\frac{1}{a * a * \cdots * a}}_n$$

$$3^2 3^{-4} = \overbrace{\cancel{3} * \cancel{3}}^2 * \underbrace{\frac{1}{\cancel{3} * \cancel{3} * 3 * 3}}_4 = \frac{1}{3 * 3} = \frac{1}{3^2} = \boxed{3^{-2}}$$

## Properties of Exponents

Definition  $a^{1/n}$

$$a^{1/n} \equiv \sqrt[n]{a} \quad \& \quad (a^n)^{1/n} = a^1 = a$$

## Properties of Exponents

Definition  $a^{1/n}$

$$a^{1/n} \equiv \sqrt[n]{a} \quad \& \quad (a^n)^{1/n} = a^1 = a$$

Why?



# Properties of Exponents

Definition  $a^{1/n}$

$$a^{1/n} \equiv \sqrt[n]{a} \quad \& \quad (a^n)^{1/n} = a^1 = a$$

Why?

$$(a^n)^{1/n} = \sqrt[n]{a^n} = a^1 = a$$

## Properties of Exponents

Definition  $a^{1/n}$

$$a^{1/n} \equiv \sqrt[n]{a} \quad \& \quad (a^n)^{1/n} = a^1 = a$$

Why?

$$(a^n)^{1/n} = \sqrt[n]{a^n} = a^1 = a$$

$$(3^4)^{1/4} = \sqrt[4]{3^4} = \boxed{3}$$

# Properties of Exponents

## Even Root $n \sqrt[n]{a}$

- $a$  positive  $\rightarrow$  result is positive.
- $a$  negative  $\rightarrow$  NOT POSSIBLE!

## Odd Root $n \sqrt[n]{a}$

- $a$  positive  $\rightarrow$  result is positive.
- $a$  negative  $\rightarrow$  result is negative.

Arithmetic examples of simplifying  
radical expressions.

Simplify the following expression

$$7\sqrt[2]{3} + 8\sqrt[2]{3} - \sqrt[2]{3}$$

Simplify the following expression

$$7\sqrt[2]{3} + 8\sqrt[2]{3} - \sqrt[2]{3}$$

$$= 7\sqrt[2]{3} + 8\sqrt[2]{3} - 1\sqrt[2]{3}$$

Simplify the following expression

$$\begin{aligned} & 7\sqrt[2]{3} + 8\sqrt[2]{3} - \sqrt[2]{3} \\ &= 7\sqrt[2]{3} + 8\sqrt[2]{3} - 1\sqrt[2]{3} \\ &= (7 + 8 - 1)\sqrt[2]{3} \end{aligned}$$

Simplify the following expression

$$\begin{aligned} & 7\sqrt[2]{3} + 8\sqrt[2]{3} - \sqrt[2]{3} \\ &= 7\sqrt[2]{3} + 8\sqrt[2]{3} - 1\sqrt[2]{3} \\ &= (7 + 8 - 1)\sqrt[2]{3} \\ &= \boxed{14\sqrt[2]{3}} \end{aligned}$$



Simplify the following expression

$$8\sqrt[3]{11} + 15\sqrt[3]{11}$$

Simplify the following expression

$$8\sqrt[3]{11} + 15\sqrt[3]{11}$$

$$= (8 + 15)\sqrt[3]{11}$$

Simplify the following expression

$$8\sqrt[3]{11} + 15\sqrt[3]{11}$$

$$= (8 + 15)\sqrt[3]{11}$$

$$= \boxed{23\sqrt[3]{11}}$$

Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9 * 7} + 9\sqrt[2]{7}$$

Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9 * 7} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9}\sqrt[2]{7} + 9\sqrt[2]{7}$$

Simplify the following expression

$$\begin{aligned} & 2\sqrt[2]{63} + 9\sqrt[2]{7} \\ &= 2\sqrt[2]{9 * 7} + 9\sqrt[2]{7} \\ &= 2\sqrt[2]{9}\sqrt[2]{7} + 9\sqrt[2]{7} \\ &= 2 * 3\sqrt[2]{7} + 9\sqrt[2]{7} \end{aligned}$$

Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9 * 7} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9}\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 2 * 3\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 6\sqrt[2]{7} + 9\sqrt[2]{7}$$



Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9 * 7} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9}\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 2 * 3\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 6\sqrt[2]{7} + 9\sqrt[2]{7} = (6 + 9)\sqrt[2]{7}$$

Simplify the following expression

$$2\sqrt[2]{63} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9 * 7} + 9\sqrt[2]{7}$$

$$= 2\sqrt[2]{9}\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 2 * 3\sqrt[2]{7} + 9\sqrt[2]{7}$$

$$= 6\sqrt[2]{7} + 9\sqrt[2]{7} = (6 + 9)\sqrt[2]{7} = \boxed{15\sqrt[2]{7}}$$

Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9 * 3} - 2\sqrt[2]{3}$$

Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9 * 3} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9}\sqrt[2]{3} - 2\sqrt[2]{3}$$

Simplify the following expression

$$\begin{aligned} & 2\sqrt[2]{27} - 2\sqrt[2]{3} \\ &= 2\sqrt[2]{9 * 3} - 2\sqrt[2]{3} \\ &= 2\sqrt[2]{9}\sqrt[2]{3} - 2\sqrt[2]{3} \\ &= 2 * 3\sqrt[2]{3} - 2\sqrt[2]{3} \end{aligned}$$

Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9 * 3} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9}\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 2 * 3\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 6\sqrt[2]{3} - 2\sqrt[2]{3}$$

Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9 * 3} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9}\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 2 * 3\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 6\sqrt[2]{3} - 2\sqrt[2]{3} = (6 - 2)\sqrt[2]{3}$$



Simplify the following expression

$$2\sqrt[2]{27} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9 * 3} - 2\sqrt[2]{3}$$

$$= 2\sqrt[2]{9}\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 2 * 3\sqrt[2]{3} - 2\sqrt[2]{3}$$

$$= 6\sqrt[2]{3} - 2\sqrt[2]{3} = (6 - 2)\sqrt[2]{3} = \boxed{4\sqrt[2]{3}}$$

Algebraic examples of simplifying  
radical expressions.

Simplify the following expression

$$\sqrt[2]{20x + 20} + \sqrt[2]{5x + 5}$$

Simplify the following expression

$$\begin{aligned} & \sqrt[2]{20x + 20} + \sqrt[2]{5x + 5} \\ &= \sqrt[2]{4(5x + 5)} + \sqrt[2]{5x + 5} \end{aligned}$$

Simplify the following expression

$$\sqrt[2]{20x + 20} + \sqrt[2]{5x + 5}$$

$$= \sqrt[2]{4(5x + 5)} + \sqrt[2]{5x + 5}$$

$$= \sqrt[2]{4} \sqrt[2]{(5x + 5)} + \sqrt[2]{5x + 5}$$

Simplify the following expression

$$\begin{aligned}& \sqrt[2]{20x + 20} + \sqrt[2]{5x + 5} \\&= \sqrt[2]{4(5x + 5)} + \sqrt[2]{5x + 5} \\&= \sqrt[2]{4} \sqrt[2]{(5x + 5)} + \sqrt[2]{5x + 5} \\&= 2 \sqrt[2]{(5x + 5)} + \sqrt[2]{5x + 5}\end{aligned}$$

Simplify the following expression

$$\begin{aligned}& \sqrt[2]{20x + 20} + \sqrt[2]{5x + 5} \\&= \sqrt[2]{4(5x + 5)} + \sqrt[2]{5x + 5} \\&= \sqrt[2]{4} \sqrt[2]{(5x + 5)} + \sqrt[2]{5x + 5} \\&= 2 \sqrt[2]{(5x + 5)} + \sqrt[2]{5x + 5} \\&= (2 + 1) \sqrt[2]{(5x + 5)}\end{aligned}$$

Simplify the following expression

$$\begin{aligned}& \sqrt[2]{20x + 20} + \sqrt[2]{5x + 5} \\&= \sqrt[2]{4(5x + 5)} + \sqrt[2]{5x + 5} \\&= \sqrt[2]{4} \sqrt[2]{(5x + 5)} + \sqrt[2]{5x + 5} \\&= 2 \sqrt[2]{(5x + 5)} + \sqrt[2]{5x + 5} \\&= (2 + 1) \sqrt[2]{(5x + 5)} = \boxed{3 \sqrt[2]{(5x + 5)}}\end{aligned}$$



Simplify the following expression

$$(\sqrt[2]{2x} + 3)(\sqrt[2]{2x} - 3)$$

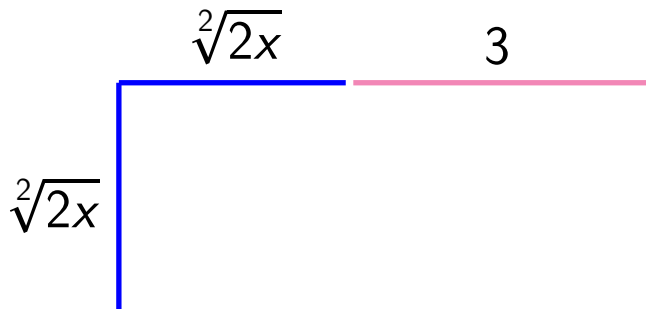
Recall how to multiply such  
quantities.

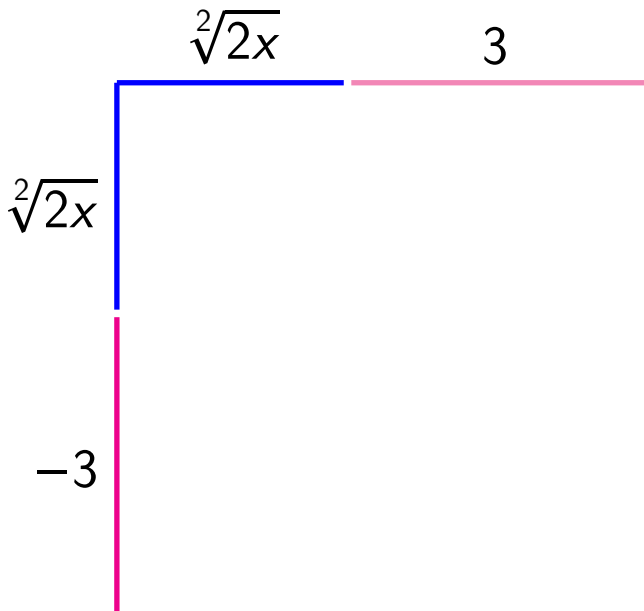
$$\sqrt{2x}$$

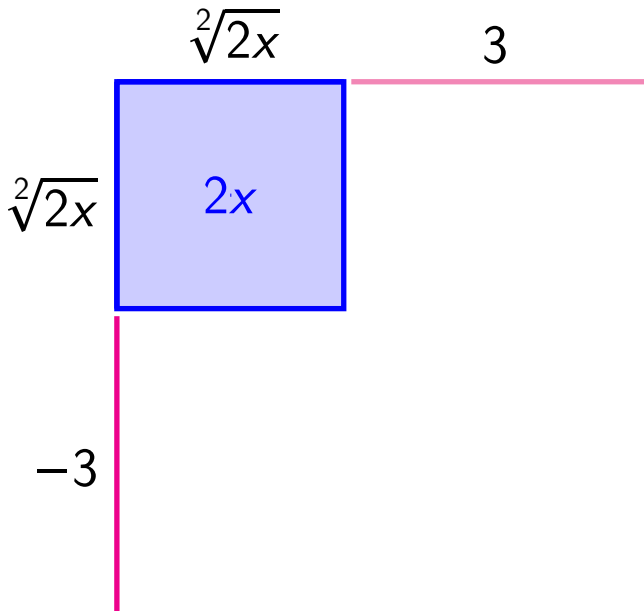


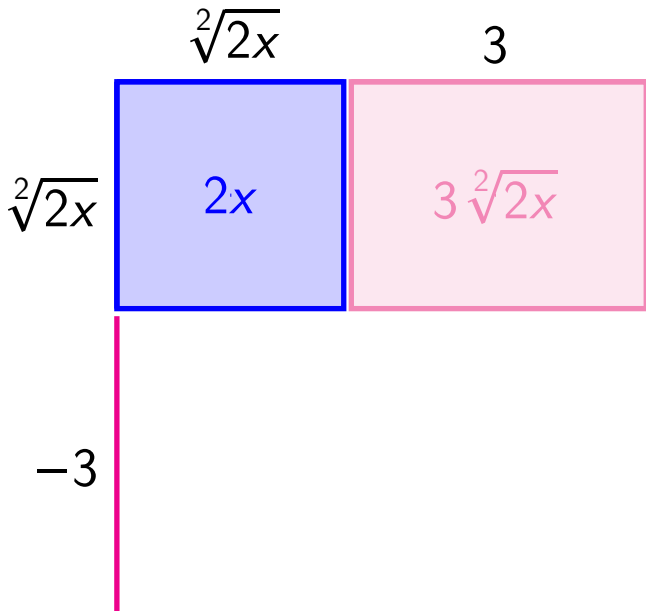
$$\sqrt[2]{2x}$$

$$3$$

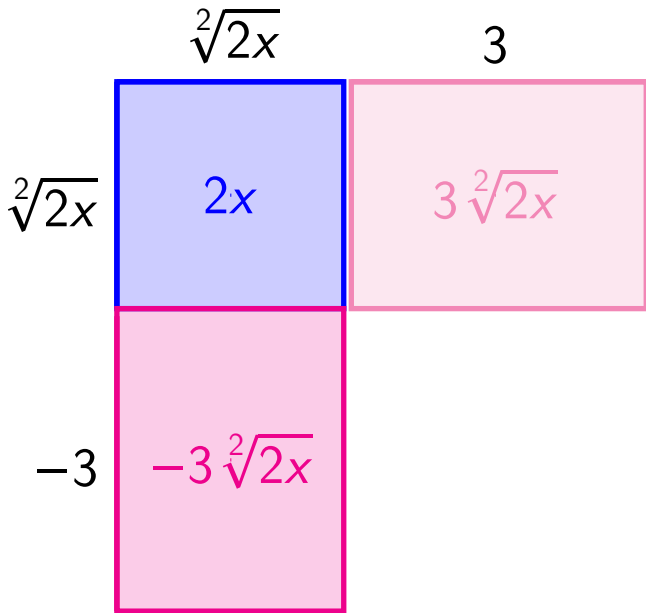




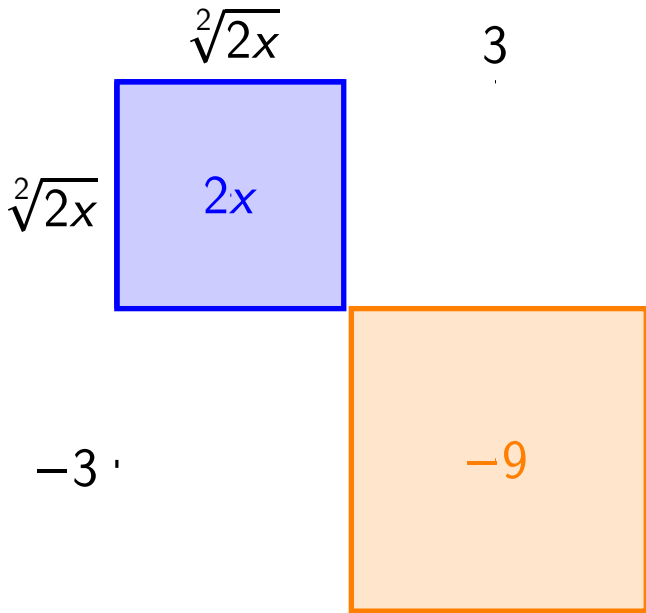








	$\sqrt[2]{2x}$	$3$
$\sqrt[2]{2x}$	$2x$	$3\sqrt[2]{2x}$
$-3$	$-3\sqrt[2]{2x}$	$-9$



Simplify the following expression

$$(\sqrt[2]{2x} + 3)(\sqrt[2]{2x} - 3)$$

Simplify the following expression

$$(\sqrt[2]{2x} + 3)(\sqrt[2]{2x} - 3)$$

$$= \sqrt[2]{2x} \sqrt[2]{2x} + 3 \sqrt[2]{2x} - 3 \sqrt[2]{2x} - 3 * 3$$

Simplify the following expression

$$(\sqrt[2]{2x} + 3)(\sqrt[2]{2x} - 3)$$

$$= \sqrt[2]{2x} \sqrt[2]{2x} + 3 \sqrt[2]{2x} - 3 \sqrt[2]{2x} - 3 * 3$$

$$= 2x + \cancel{3\sqrt[2]{2x}} - \cancel{3\sqrt[2]{2x}} - 9$$

Simplify the following expression

$$(\sqrt[2]{2x} + 3)(\sqrt[2]{2x} - 3)$$

$$= \sqrt[2]{2x} \sqrt[2]{2x} + 3 \sqrt[2]{2x} - 3 \sqrt[2]{2x} - 3 * 3$$

$$= 2x + \cancel{3\sqrt[2]{2x}} - \cancel{3\sqrt[2]{2x}} - 9$$

$$= \boxed{2x - 9}$$

Simplify the following expression

$$(3\sqrt[2]{x} + \sqrt[2]{y})(3\sqrt[2]{x} - \sqrt[2]{y})$$



$$3\sqrt[2]{x}$$

---

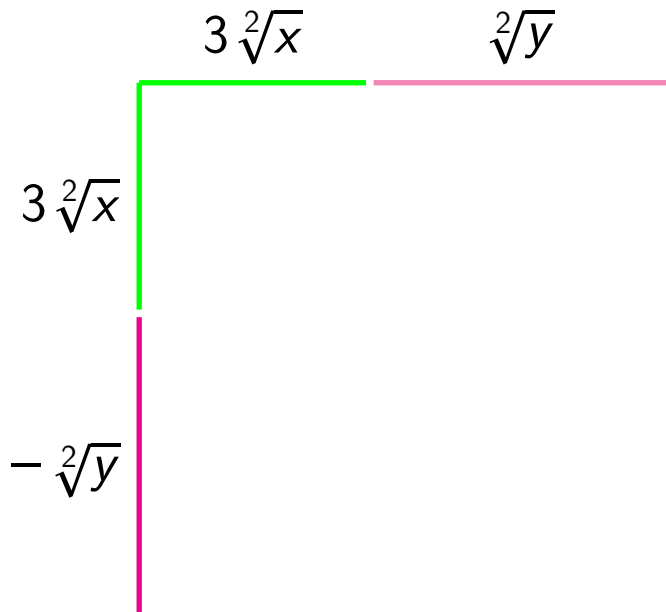
$$3\sqrt[2]{x}$$

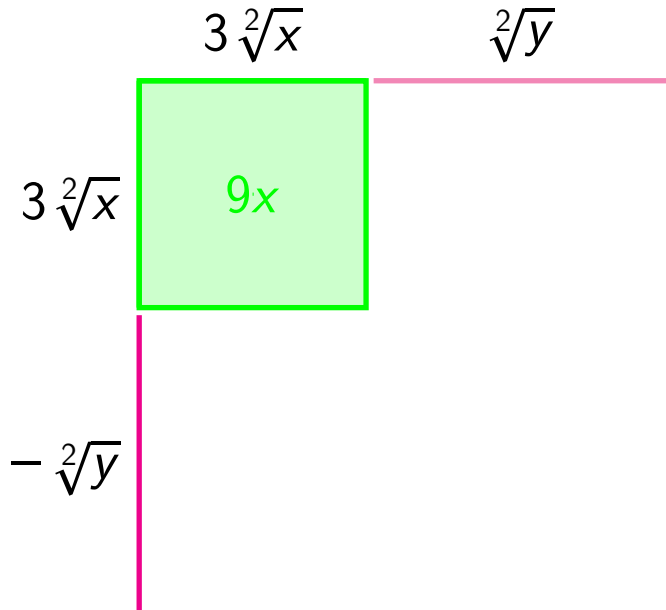
$$\sqrt[2]{y}$$



A diagram illustrating a path or boundary. It consists of a vertical green line segment on the left and a horizontal pink line segment on the right, meeting at a corner. The expression  $3\sqrt[2]{x}$  is placed to the left of the vertical segment. The expression  $3\sqrt[2]{x}$  is placed above the horizontal segment, and the expression  $\sqrt[2]{y}$  is placed above the horizontal segment to the right of the corner.

$$3\sqrt[2]{x}$$
$$3\sqrt[2]{x}$$
$$\sqrt[2]{y}$$





$$3\sqrt[2]{x}$$

$$\sqrt[2]{y}$$

$$3\sqrt[2]{x}$$

$$9x$$

$$3\sqrt[2]{x}\sqrt[2]{y}$$

$$-\sqrt[2]{y}$$

$$3\sqrt[2]{x}$$

$$\sqrt[2]{y}$$

$$3\sqrt[2]{x}$$

$$9x$$

$$3\sqrt[2]{x}\sqrt[2]{y}$$

$$-\sqrt[2]{y}$$

$$-3\sqrt[2]{x}\sqrt[2]{y}$$

$$3\sqrt[2]{x}$$

$$\sqrt[2]{y}$$

$$3\sqrt[2]{x}$$

$$9x$$

$$3\sqrt[2]{x}\sqrt[2]{y}$$

$$-\sqrt[2]{y}$$

$$-3\sqrt[2]{x}\sqrt[2]{y}$$

$$-y$$



$$3\sqrt[2]{x}$$

$$\sqrt[2]{y}$$

$$3\sqrt[2]{x}$$

$$9x$$

$$-\sqrt[2]{y}$$

$$-y$$

Simplify the following expression

$$(3\sqrt[2]{x} + \sqrt[2]{y})(3\sqrt[2]{x} - \sqrt[2]{y})$$

Simplify the following expression

$$(3\sqrt[2]{x} + \sqrt[2]{y})(3\sqrt[2]{x} - \sqrt[2]{y})$$

$$= 3\sqrt[2]{x} * 3\sqrt[2]{x} +$$

$$\sqrt[2]{y} * 3\sqrt[2]{x} - 3\sqrt[2]{x} * \sqrt[2]{y} - \sqrt[2]{y} * \sqrt[2]{y}$$

Simplify the following expression

$$(3\sqrt[2]{x} + \sqrt[2]{y})(3\sqrt[2]{x} - \sqrt[2]{y})$$

$$= 3\sqrt[2]{x} * 3\sqrt[2]{x} +$$

$$\sqrt[2]{y} * 3\sqrt[2]{x} - 3\sqrt[2]{x} * \sqrt[2]{y} - \sqrt[2]{y} * \sqrt[2]{y}$$

$$= 9x + \cancel{\sqrt[2]{y} * 3\sqrt[2]{x}} - \cancel{3\sqrt[2]{x} * \sqrt[2]{y}} - y$$

Simplify the following expression

$$(3\sqrt[2]{x} + \sqrt[2]{y})(3\sqrt[2]{x} - \sqrt[2]{y})$$

$$= 3\sqrt[2]{x} * 3\sqrt[2]{x} +$$

$$\sqrt[2]{y} * 3\sqrt[2]{x} - 3\sqrt[2]{x} * \sqrt[2]{y} - \sqrt[2]{y} * \sqrt[2]{y}$$

$$= 9x + \cancel{\sqrt[2]{y} * 3\sqrt[2]{x}} - \cancel{3\sqrt[2]{x} * \sqrt[2]{y}} - y$$

$$= \boxed{9x - y}$$

Rationalizing a fraction with a radical expression in the denominator.

## Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}}$$

## Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}} \frac{\sqrt[2]{7}}{\sqrt[2]{7}}$$



## Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}} \frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}}$$

## Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}} \frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}} = \boxed{\frac{2\sqrt[2]{7}}{7}}$$

## Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}} \frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}} = \boxed{\frac{2\sqrt[2]{7}}{7}}$$

$$\frac{8}{\sqrt[2]{2}}$$

## Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}} \frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}} = \boxed{\frac{2\sqrt[2]{7}}{7}}$$

$$\frac{8}{\sqrt[2]{2}} = \frac{8}{\sqrt[2]{2}} \frac{\sqrt[2]{2}}{\sqrt[2]{2}}$$

## Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}} \frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}} = \boxed{\frac{2\sqrt[2]{7}}{7}}$$

$$\frac{8}{\sqrt[2]{2}} = \frac{8}{\sqrt[2]{2}} \frac{\sqrt[2]{2}}{\sqrt[2]{2}} = \frac{8\sqrt[2]{2}}{\sqrt[2]{2}\sqrt[2]{2}}$$

## Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}} \frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}} = \boxed{\frac{2\sqrt[2]{7}}{7}}$$

$$\frac{8}{\sqrt[2]{2}} = \frac{8}{\sqrt[2]{2}} \frac{\sqrt[2]{2}}{\sqrt[2]{2}} = \frac{8\sqrt[2]{2}}{\sqrt[2]{2}\sqrt[2]{2}} = \frac{8\sqrt[2]{2}}{2}$$

## Rationalizing the Denominator

$$\frac{2}{\sqrt[2]{7}} = \frac{2}{\sqrt[2]{7}} \frac{\sqrt[2]{7}}{\sqrt[2]{7}} = \frac{2\sqrt[2]{7}}{\sqrt[2]{7}\sqrt[2]{7}} = \boxed{\frac{2\sqrt[2]{7}}{7}}$$

$$\begin{aligned}\frac{8}{\sqrt[2]{2}} &= \frac{8}{\sqrt[2]{2}} \frac{\sqrt[2]{2}}{\sqrt[2]{2}} = \frac{8\sqrt[2]{2}}{\sqrt[2]{2}\sqrt[2]{2}} = \frac{8\sqrt[2]{2}}{2} \\ &= \boxed{4\sqrt[2]{2}}\end{aligned}$$

Solving radical equations.



Solve for the unknown variable ♥

$$\sqrt[2]{\heartsuit} = 4$$

Solve for the unknown variable ♥

$$\sqrt[2]{\heartsuit} = 4$$

$$\left(\sqrt[2]{\heartsuit}\right)^2 = 4^2$$

Solve for the unknown variable ♥

$$\sqrt[2]{\heartsuit} = 4$$

$$\left(\sqrt[2]{\heartsuit}\right)^2 = 4^2$$

$$\left(\sqrt[2]{\heartsuit}\right)^2 = 16$$

Solve for the unknown variable ♥

$$\sqrt[2]{\heartsuit} = 4$$

$$\left(\sqrt[2]{\heartsuit}\right)^2 = 4^2$$

$$\left(\sqrt[2]{\heartsuit}\right)^2 = 16$$

$$\boxed{\heartsuit = 16}$$

Solve for the unknown variable ♦

$$\sqrt[2]{3\diamond + 7} = 5$$

Solve for the unknown variable ♦

$$\sqrt[2]{3\diamond + 7} = 5$$

$$\left(\sqrt[2]{3\diamond + 7}\right)^2 = 5^2$$

Solve for the unknown variable ♦

$$\sqrt[2]{3\diamond + 7} = 5$$

$$\left(\sqrt[2]{3\diamond + 7}\right)^2 = 5^2$$

$$3\diamond + 7 = 25$$

Solve for the unknown variable ♦

$$\sqrt[2]{3\diamond + 7} = 5$$

$$\left(\sqrt[2]{3\diamond + 7}\right)^2 = 5^2$$

$$3\diamond + 7 = 25$$

$$3\diamond = 18$$



Solve for the unknown variable ♦

$$\sqrt[2]{3\diamond + 7} = 5$$

$$\left(\sqrt[2]{3\diamond + 7}\right)^2 = 5^2$$

$$3\diamond + 7 = 25$$

$$3\diamond = 18$$

$$\boxed{\diamond = 6}$$

Solve for the unknown variable ♣

$$\sqrt[3]{\clubsuit} = -4$$

Solve for the unknown variable ♣

$$\sqrt[3]{\clubsuit} = -4$$

$$\left(\sqrt[3]{\clubsuit}\right)^3 = -4^3$$

Solve for the unknown variable ♣

$$\sqrt[3]{\clubsuit} = -4$$

$$\left(\sqrt[3]{\clubsuit}\right)^3 = -4^3$$

$$\left(\cancel{\sqrt[3]}{\clubsuit}\right)^{\cancel{3}} = -4^3$$

Solve for the unknown variable ♣

$$\sqrt[3]{\clubsuit} = -4$$

$$\left(\sqrt[3]{\clubsuit}\right)^3 = -4^3$$

$$\left(\cancel{\sqrt[3]}{\clubsuit}\right)^{\cancel{3}} = -4^3$$

$$\boxed{\clubsuit = -64}$$

Solve for the unknown variable ♥

$$\heartsuit = \sqrt[2]{\heartsuit + 6}$$

Solve for the unknown variable ♡

$$\heartsuit = \sqrt[2]{\heartsuit + 6}$$

$$\heartsuit^2 = \left( \sqrt[2]{\heartsuit + 6} \right)^2$$

Solve for the unknown variable ♡

$$♡ = \sqrt[2]{♡ + 6}$$

$$♡^2 = \left( \sqrt[2]{♡ + 6} \right)^2$$

$$♡^2 = \left( \sqrt[2]{♡ + 6} \right)^2$$



Solve for the unknown variable ♥

$$\heartsuit^2 = \heartsuit + 6$$

Solve for the unknown variable ♥

$$\heartsuit^2 = \heartsuit + 6$$

$$\heartsuit^2 - \heartsuit - 6 = 0$$

Solve for the unknown variable ♥

$$\heartsuit^2 = \heartsuit + 6$$

$$\heartsuit^2 - \heartsuit - 6 = 0$$

$$(\heartsuit - 3)(\heartsuit + 2) = 0$$

Solve for the unknown variable ♥

$$\heartsuit^2 = \heartsuit + 6$$

$$\heartsuit^2 - \heartsuit - 6 = 0$$

$$(\heartsuit - 3)(\heartsuit + 2) = 0$$

$$\boxed{\heartsuit = -3}$$

Next Time

Applications of Radical Expressions

Copyright 2016 Crista Moreno. Algebra Lecture 21 is made available under the Creative Commons Attribution-ShareAlike 4.0 International License.

To view a copy of this license, visit

<http://creativecommons.org/licenses/by-sa/4.0/>.

