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Algebra Lecture 16

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December 19, 2016

Last Time

- Special Cases of Factoring
- Polynomial Equations

Topics for Today

- Rational Functions
- Domains of Rational Functions
- Multiplying and Dividing Rational Expressions

Recall what is a **function**.

Definition of a Function

A relation in which each x -coordinate is matched with only one y -coordinate is said to describe y as a **function** of x .

Recall what is the **domain** of a
function.

Domain

The **domain** of a function consists of the **x -coordinates**.

What is a Rational Function?

A rational function is a ratio of two polynomial functions.

Form of a Rational Function

$$R(x) = \frac{P_1(x)}{P_2(x)}, \text{ where } P_2(x) \neq 0$$

Is the following Function a Rational Function?

$$f(x) = \frac{x^2 + 4x}{2x}$$

Is the following Function a Rational Function?

$$f(x) = \frac{x^2 + 4x}{2x}$$

What are the two polynomial functions?

$$p_1(x) = x^2 + 4x$$

$$p_2(x) = 2x$$

Is the following Function a Rational Function?

$$f(x) = \frac{1}{3x^3}$$

$$p_1(x) = 1$$

$$p_2(x) = 3x^3$$

Is the following Function a Rational Function?

$$f(x) = \log(x^3)$$

Is the following Function a Rational Function?

$$f(x) = \log(x^3)$$

No! This is a logarithmic function.

Is the following Function a Rational Function?

$$f(x) = 5x^4 + 3x^3 + 2x^2 + 1$$

$$p_1(x) = 5x^4 + 3x^3 + 2x^2 + 1$$

$$p_2(x) = 1$$

All polynomial functions are rational functions, and only rational functions with a constant function as the denominator are polynomial functions.

What is the domain of the rational function $f(x)$?

Consider the following rational function

$$f(x) = \frac{1}{x + 1}$$

What is the domain of the rational function $f(x)$?

Consider the following rational function

$$f(x) = \frac{1}{x + 1}$$

What is the domain of this function?

Domain of Rational Functions

Domain of Rational Functions

- We know that a rational function takes the form of a **fraction**.

Domain of Rational Functions

- We know that a rational function takes the form of a **fraction**.
- What can **never** happen with a fraction?

Domain of Rational Functions

- We know that a rational function takes the form of a **fraction**.
- What can **never** happen with a fraction?
- The denominator of a fraction can **never** be **zero**.

What is the domain of the rational function $f(x)$?

$$f(x) = \frac{1}{x + 1}$$

What is the domain of the rational function $f(x)$?

$$f(x) = \frac{1}{x + 1}$$

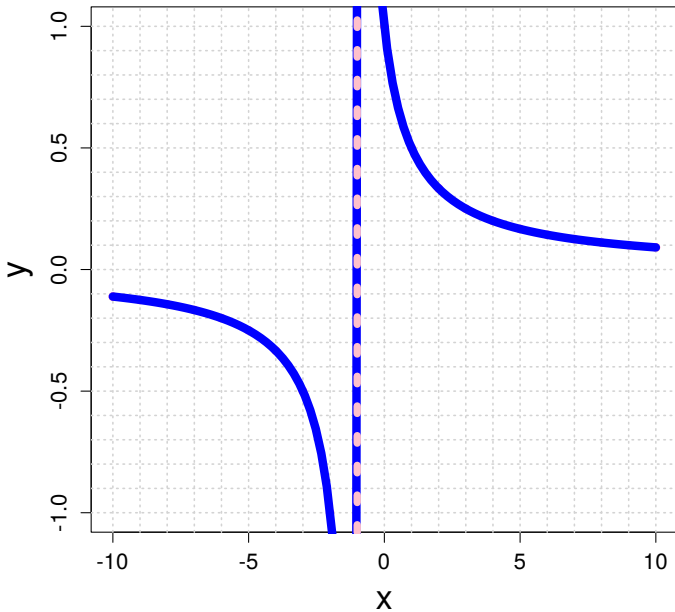
- $x + 1 \neq 0$

What is the domain of the rational function $f(x)$?

$$f(x) = \frac{1}{x+1}$$

- $x + 1 \neq 0$
- $\{x \mid x \neq -1\}$ or $(-\infty, -1) \cup (-1, \infty)$

$$f(x) = 1/(x + 1)$$

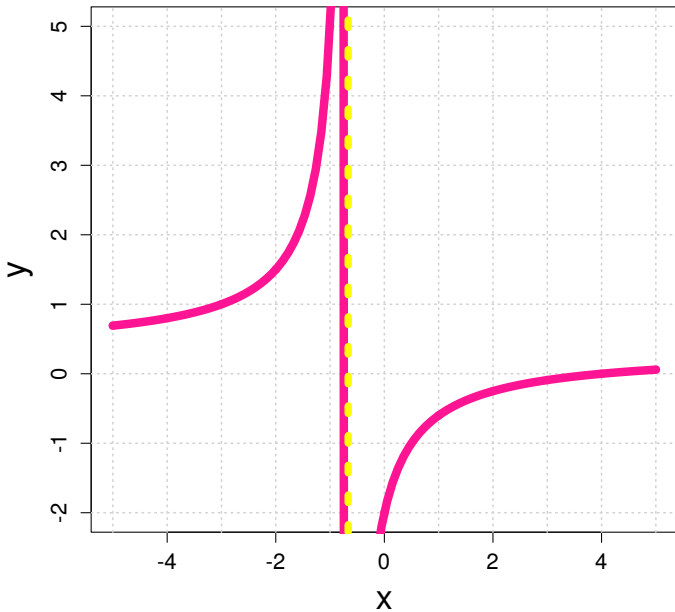


Domains of Rational Functions

$$f(x) = \frac{x - 4}{3x + 2}$$

- Find $f(-3)$
- Find $f(0)$
- Find $f(-2/3)$
- Give the domain in set notation.

$$f(x) = (x - 4)/(3x + 2)$$



Domains of Rational Functions

$$f(x) = \frac{x - 4}{3x + 2}$$

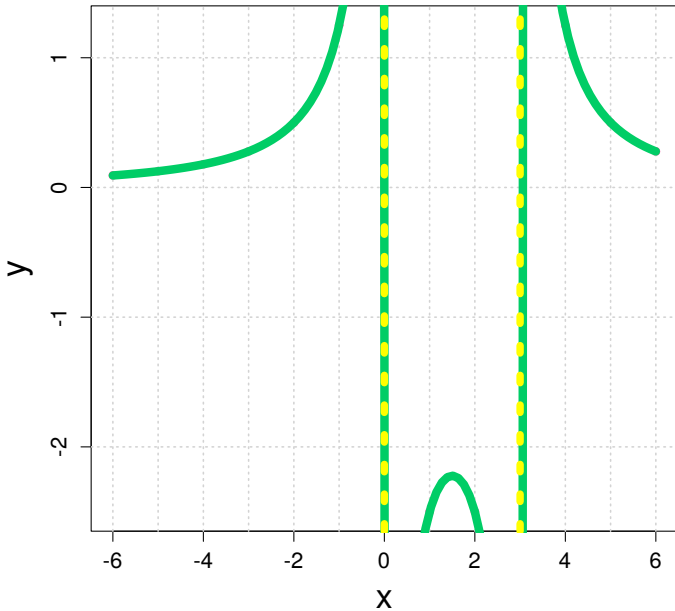
- Find $f(-3) = 1$
- Find $f(0) = -2$
- Find $f(-2/3) = \text{Not Defined}$
- Give the domain in set notation. $\{x|x \neq -2/3\}$

Domains of Rational Functions

$$f(x) = \frac{5}{x^2 - 3x}$$

- Find $f(-2)$
- Give the domain in set notation.

$$f(x) = 5/(x^2 - 3x)$$



Domains of Rational Functions

$$f(x) = \frac{5}{x^2 - 3x}$$

- Find $f(-2) = \frac{1}{2}$
- Give the domain in set notation.

$$\{x \mid x \neq 0, x \neq 3\}$$

Solving Rational Equations

Solve the following Rational Equation

$$\frac{8}{\heartsuit} = 5$$

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$$\heartsuit \frac{8}{\heartsuit} = 5 \heartsuit$$

Solve the following Rational Equation

$$\frac{8}{\heartsuit} = 5$$

$$\heartsuit \frac{8}{\heartsuit} = 5 \heartsuit$$

$$\cancel{\heartsuit} \frac{8}{\cancel{\heartsuit}} = 5 \heartsuit$$

Solve the following Rational Equation

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$$\cancel{\heartsuit} \frac{8}{\cancel{\heartsuit}} = 5 \heartsuit$$

$$8 = 5 \heartsuit$$

Solve the following Rational Equation

$$\frac{8}{\heartsuit} = 5$$

$$\heartsuit \frac{8}{\heartsuit} = 5 \heartsuit$$

$$\cancel{\heartsuit} \frac{8}{\cancel{\heartsuit}} = 5 \heartsuit$$

$$8 = 5 \heartsuit$$

$$\boxed{\heartsuit = \frac{8}{5}}$$

Solve the following Rational Equation

$$\frac{x}{x+16} - 3 = 2$$

Solve the following Rational Equation

$$\frac{x}{x+16} - 3 = 2$$

$$\frac{x}{x+16} = 5$$

Solve the following Rational Equation

$$\frac{x}{x+16} - 3 = 2$$

$$\frac{x}{x+16} = 5$$

$$(x+16)\frac{x}{x+16} = 5(x+16)$$

Solve the following Rational Equation

$$\frac{x}{x+16} - 3 = 2$$

$$\frac{x}{x+16} = 5$$

$$(x+16)\frac{x}{x+16} = 5(x+16)$$

$$\cancel{(x+16)}\frac{x}{\cancel{(x+16)}} = 5(x+16)$$

Solve the following Rational Equation

$$x = 5(x + 16)$$

Solve the following Rational Equation

$$x = 5(x + 16)$$

$$x = 5x + 80$$

Solve the following Rational Equation

$$x = 5(x + 16)$$

$$x = 5x + 80$$

$$-4x = 80$$

Solve the following Rational Equation

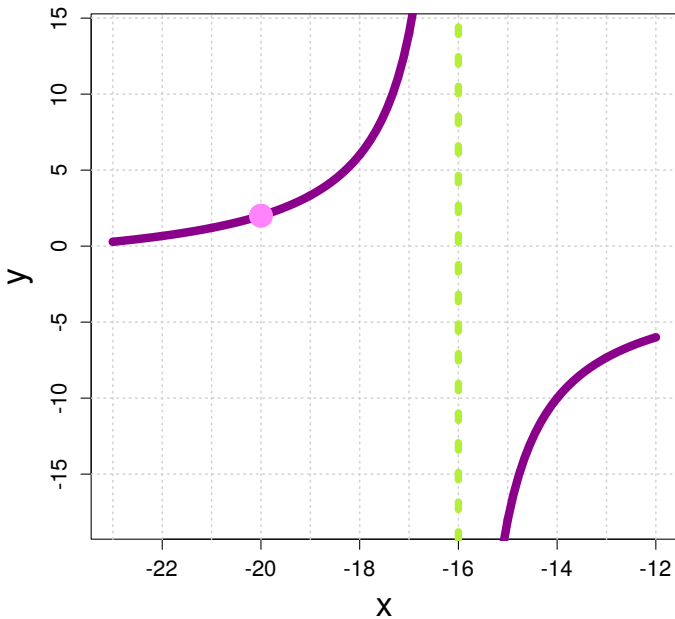
$$x = 5(x + 16)$$

$$x = 5x + 80$$

$$-4x = 80$$

$$x = -20$$

$$f(x) = x/(x + 16) - 3$$



Solve the following Rational Equation

$$\frac{2x + 1}{3x - 2} = 1$$

Solve the following Rational Equation

$$\frac{2x + 1}{3x - 2} = 1$$

$$(3x - 2) \frac{2x + 1}{3x - 2} = 1(3x - 2)$$

Solve the following Rational Equation

$$\frac{2x + 1}{3x - 2} = 1$$

$$(3x - 2) \frac{2x + 1}{3x - 2} = 1(3x - 2)$$

$$\cancel{(3x - 2)} \frac{2x + 1}{\cancel{(3x - 2)}} = 1(3x - 2)$$

Solve the following Rational Equation

$$\frac{2x + 1}{3x - 2} = 1$$

$$(3x - 2) \frac{2x + 1}{3x - 2} = 1(3x - 2)$$

$$\cancel{(3x - 2)} \frac{2x + 1}{\cancel{(3x - 2)}} = 1(3x - 2)$$

$$2x + 1 = 1(3x - 2)$$

Solve the following Rational Equation

$$2x + 1 = 1(3x - 2)$$

Solve the following Rational Equation

$$2x + 1 = 1(3x - 2)$$

$$2x + 1 = 3x - 2$$

Solve the following Rational Equation

$$2x + 1 = 1(3x - 2)$$

$$2x + 1 = 3x - 2$$

$$2x = 3x - 3$$

Solve the following Rational Equation

$$2x + 1 = 1(3x - 2)$$

$$2x + 1 = 3x - 2$$

$$2x = 3x - 3$$

$$-x = -3$$

Solve the following Rational Equation

$$2x + 1 = 1(3x - 2)$$

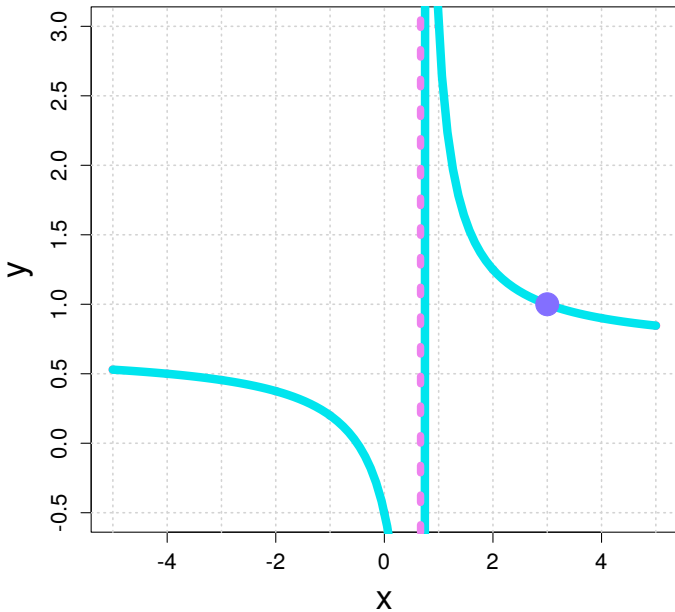
$$2x + 1 = 3x - 2$$

$$2x = 3x - 3$$

$$-x = -3$$

$$x = 3$$

$$f(x) = (2x + 1)/(3x - 2)$$



Solve the following Rational Equation

$$\frac{x}{x+1} = \frac{2x+1}{x+1}$$

Solve the following Rational Equation

$$\frac{x}{x+1} = \frac{2x+1}{x+1}$$

$$x = \frac{2x+1}{x+1}(x+1)$$

Solve the following Rational Equation

$$\frac{x}{x+1} = \frac{2x+1}{x+1}$$

$$x = \frac{2x+1}{x+1}(x+1)$$

$$x = \frac{2x+1}{\cancel{x+1}}(\cancel{x+1})$$

Solve the following Rational Equation

$$\frac{x}{x+1} = \frac{2x+1}{x+1}$$

$$x = \frac{2x+1}{x+1}(x+1)$$

$$x = \frac{2x+1}{\cancel{x+1}}(\cancel{x+1})$$

$$x = 2x + 1$$

Solve the following Rational Equation

$$\frac{x}{x+1} = \frac{2x+1}{x+1}$$

$$x = \frac{2x+1}{x+1}(x+1)$$

$$x = \frac{2x+1}{\cancel{x+1}}(\cancel{x+1})$$

$$x = 2x + 1$$

$$-x = 1$$

Solve the following Rational Equation

$$\frac{x}{x+1} = \frac{2x+1}{x+1}$$

$$x = \frac{2x+1}{x+1}(x+1)$$

$$x = \frac{2x+1}{\cancel{x+1}}(\cancel{x+1})$$

$$x = 2x + 1$$

$$-x = 1$$

$x = -1$ Not possible!!

Solve the following Rational Equation

$$\frac{x}{x+1} = \frac{2x+1}{x+1}$$

$$x = \frac{2x+1}{x+1}(x+1)$$

$$x = \frac{2x+1}{\cancel{x+1}}(\cancel{x+1})$$

$$x = 2x + 1$$

$$-x = 1$$

$x = -1$ Not possible!! $x = \emptyset$

Solve the following Rational Equation

$$\frac{4}{x-3} = x$$

Solve the following Rational Equation

$$\frac{4}{x-3} = x$$

$$4 = x(x-3)$$

Solve the following Rational Equation

$$\frac{4}{x-3} = x$$

$$4 = x(x-3)$$

$$4 = x^2 - 3x$$

Solve the following Rational Equation

$$\frac{4}{x-3} = x$$

$$4 = x(x-3)$$

$$4 = x^2 - 3x$$

$$0 = x^2 - 3x - 4$$

Solve the following Rational Equation

$$\frac{4}{x-3} = x$$

$$4 = x(x-3)$$

$$4 = x^2 - 3x$$

$$0 = x^2 - 3x - 4$$

$$0 = (x-4)(x+1)$$

Solve the following Rational Equation

$$\frac{4}{x-3} = x$$

$$4 = x(x-3)$$

$$4 = x^2 - 3x$$

$$0 = x^2 - 3x - 4$$

$$0 = (x-4)(x+1)$$

$$x = 4 \text{ or } x = -1$$

Solve the following Rational Equation

$$\frac{4}{x-3} = x$$

$$4 = x(x-3)$$

$$4 = x^2 - 3x$$

$$0 = x^2 - 3x - 4$$

$$0 = (x-4)(x+1)$$

$$x = 4 \text{ or } x = -1 \quad \boxed{x = \{4, -1\}}$$

Multiplying and Dividing Rational Expressions

Simplify the Rational Expression

$$\frac{x + 5}{x^2 + 2x - 15}$$

Simplify the Rational Expression

$$\frac{x + 5}{x^2 + 2x - 15}$$
$$= \frac{x + 5}{(x + 5)(x - 3)}$$

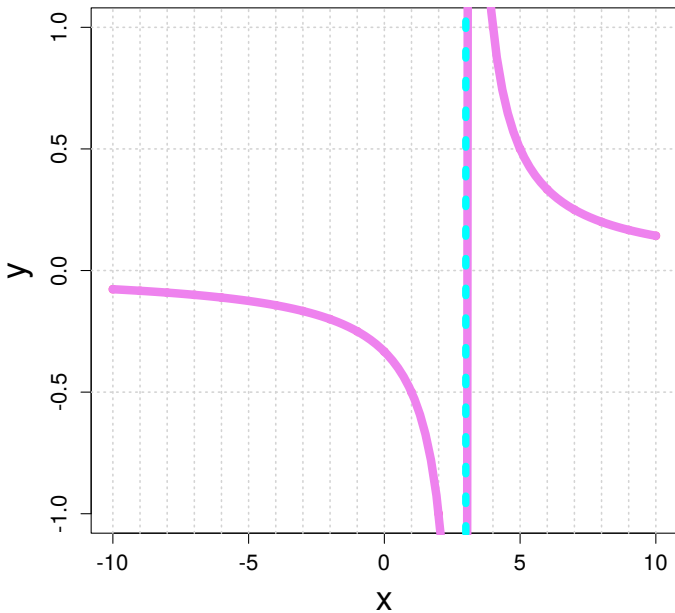
Simplify the Rational Expression

$$\begin{aligned}& \frac{x+5}{x^2+2x-15} \\&= \frac{x+5}{(x+5)(x-3)} \\&= \frac{\cancel{(x+5)}}{\cancel{(x+5)}(x-3)}\end{aligned}$$

Simplify the Rational Expression

$$\begin{aligned}& \frac{x+5}{x^2+2x-15} \\&= \frac{x+5}{(x+5)(x-3)} \\&= \frac{\cancel{(x+5)}}{\cancel{(x+5)}(x-3)} \\&= \boxed{\frac{1}{(x-3)}}$$

$$f(x) = (x + 5)/(x^2 + 2x - 15)$$



Simplify the Rational Expression

$$\frac{6x^2 + 7x - 5}{2x^2 - 11x + 5}$$

Simplify the Rational Expression

$$\frac{6x^2 + 7x - 5}{2x^2 - 11x + 5}$$
$$= \frac{(3x + 5)(2x - 1)}{(2x - 1)(x - 5)}$$

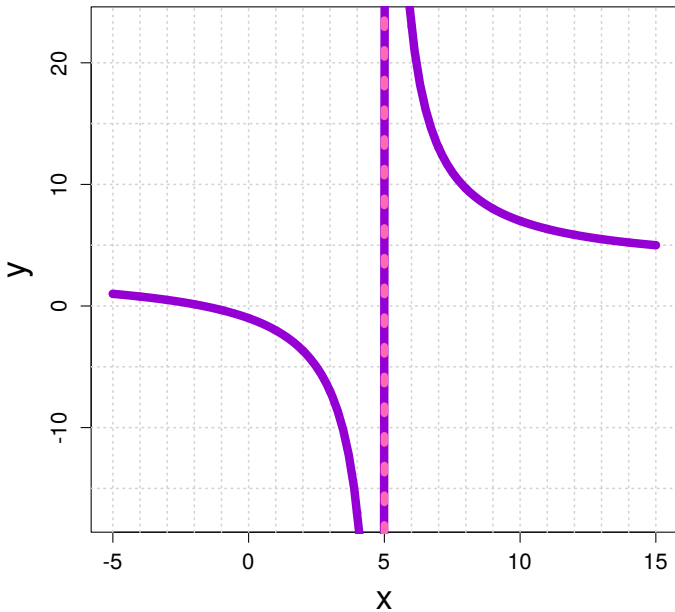
Simplify the Rational Expression

$$\begin{aligned}& \frac{6x^2 + 7x - 5}{2x^2 - 11x + 5} \\&= \frac{(3x + 5)(2x - 1)}{(2x - 1)(x - 5)} \\&= \frac{(3x + 5)\cancel{(2x - 1)}}{\cancel{(2x - 1)}(x - 5)}\end{aligned}$$

Simplify the Rational Expression

$$\begin{aligned}& \frac{6x^2 + 7x - 5}{2x^2 - 11x + 5} \\&= \frac{(3x + 5)(2x - 1)}{(2x - 1)(x - 5)} \\&= \frac{(3x + 5)\cancel{(2x - 1)}}{\cancel{(2x - 1)}(x - 5)} \\&= \boxed{\frac{(3x + 5)}{(x - 5)}}$$

$$f(x) = (6x^2 + 7x - 5)/(2x^2 - 11x + 5)$$



Simplify the Rational Expression

$$= \frac{x+5}{x^3-x} \div \frac{x^2-25}{x^4}$$

Simplify the Rational Expression

$$= \frac{x+5}{x^3-x} \div \frac{x^2-25}{x^4}$$

$$= \frac{x+5}{x^3-x} \times \frac{x^4}{x^2-25}$$

Simplify the Rational Expression

$$= \frac{x+5}{x^3-x} \div \frac{x^2-25}{x^4}$$

$$= \frac{x+5}{x^3-x} \times \frac{x^4}{x^2-25}$$

$$= \frac{x+5}{x(x^2-1)} \times \frac{x^4}{x^2-25}$$

Simplify the Rational Expression

$$= \frac{x+5}{x^3-x} \div \frac{x^2-25}{x^4}$$

$$= \frac{x+5}{x^3-x} \times \frac{x^4}{x^2-25}$$

$$= \frac{x+5}{x(x^2-1)} \times \frac{x^4}{x^2-25}$$

$$= \frac{x+5}{x(x^2-1)} \times \frac{x^4}{(x+5)(x-5)}$$

Simplify the Rational Expression

$$= \frac{x+5}{x^3-x} \div \frac{x^2-25}{x^4}$$

$$= \frac{x+5}{x^3-x} \times \frac{x^4}{x^2-25}$$

$$= \frac{x+5}{x(x^2-1)} \times \frac{x^4}{x^2-25}$$

$$= \frac{x+5}{x(x^2-1)} \times \frac{x^4}{(x+5)(x-5)}$$

$$= \frac{x+5}{x(x+1)(x-1)} \times \frac{x^4}{(x+5)(x-5)}$$

Simplify the Rational Expression

$$= \frac{\cancel{(x+5)}}{x(x+1)(x-1)} \times \frac{x^4}{\cancel{(x+5)}(x-5)}$$

Simplify the Rational Expression

$$= \frac{\cancel{(x+5)}}{x(x+1)(x-1)} \times \frac{x^4}{\cancel{(x+5)}(x-5)}$$

$$= \frac{1}{x(x+1)(x-1)} \times \frac{x^4}{(x-5)}$$

Simplify the Rational Expression

$$= \frac{\cancel{(x+5)}}{x(x+1)(x-1)} \times \frac{x^4}{\cancel{(x+5)}(x-5)}$$

$$= \frac{1}{x(x+1)(x-1)} \times \frac{x^4}{(x-5)}$$

$$= \frac{1}{\cancel{x}(x+1)(x-1)} \times \frac{x^{\cancel{4}}}{(x-5)}$$

Simplify the Rational Expression

$$= \frac{\cancel{(x+5)}}{x(x+1)(x-1)} \times \frac{x^4}{\cancel{(x+5)}(x-5)}$$

$$= \frac{1}{x(x+1)(x-1)} \times \frac{x^4}{(x-5)}$$

$$= \frac{1}{\cancel{x}(x+1)(x-1)} \times \frac{x^{\cancel{4}}}{(x-5)}$$

$$= \frac{1}{(x+1)(x-1)} \times \frac{x^3}{(x-5)}$$

Simplify the Rational Expression

$$= \frac{\cancel{(x+5)}}{x(x+1)(x-1)} \times \frac{x^4}{\cancel{(x+5)}(x-5)}$$

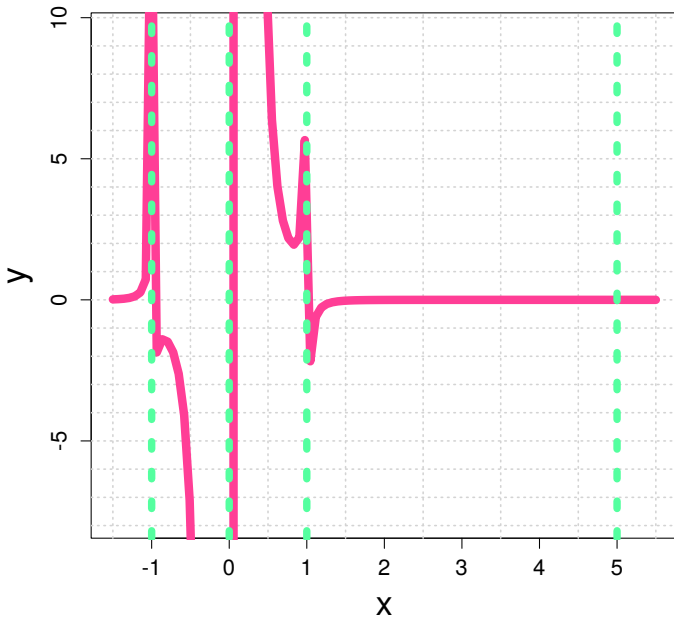
$$= \frac{1}{x(x+1)(x-1)} \times \frac{x^4}{(x-5)}$$

$$= \frac{1}{\cancel{x}(x+1)(x-1)} \times \frac{x^{\cancel{4}}}{(x-5)}$$

$$= \frac{1}{(x+1)(x-1)} \times \frac{x^3}{(x-5)}$$

$$\boxed{\frac{x^3}{(x+1)(x-1)(x-5)}}$$

$$f(x) = \frac{x^3}{(x+1)(x-1)(x-5)}$$



Next Time

Adding and Subtracting Rational Expressions
& Solving Rational Equations

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