Algebra Lecture 15

Crista Moreno

December 19, 2016

Topics

Last Time

- Factoring Trinomials

Topics

Topics for Today

- Special Cases of Factoring

  & Polynomial Equations

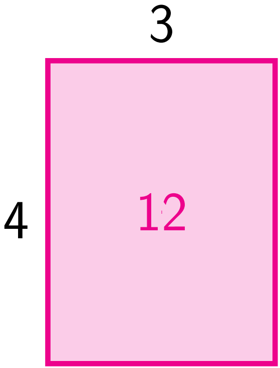
Recall what it means to multiply two quantities.

Geometrically, we think of multiplication as the **area** of some rectangle.

What does the multiplication $3 * 4$

mean geometrically?

Multiplication $3 * 4$



What does the multiplication

$(2 + x) * 3$ mean geometrically?

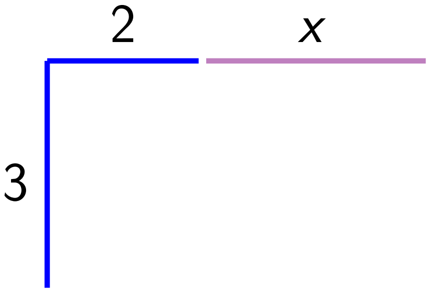
Multiplication $3 * (2 + x)$

$$\frac{2}{}$$

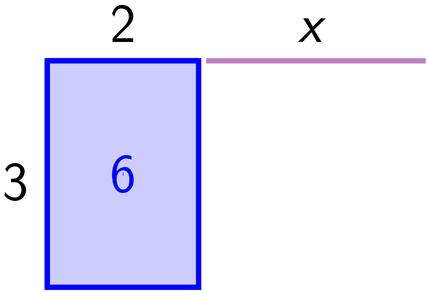
Multiplication $3 * (2 + x)$

$$\underline{\quad 2 \quad}\ \underline{\quad x \quad}$$

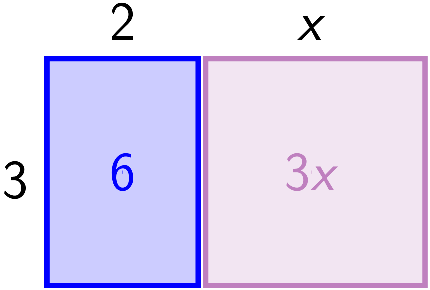
Multiplication $3 * (2 + x)$



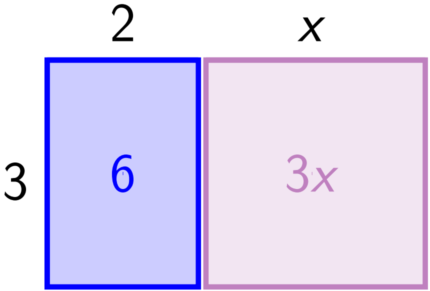
Multiplication $3 * (2 + x)$



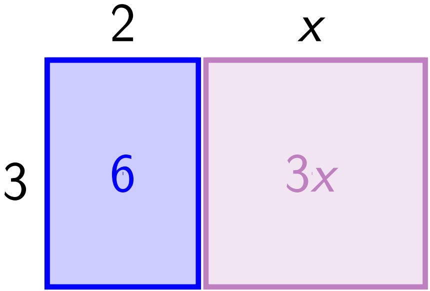
Multiplication $3 * (2 + x)$



Multiplication $3 * (2 + x)$



Multiplication $3 * (2 + x)$



This gives the area $3 * 2 + 3 * x$ or $6 + 3x$, which is what the distributive property would give.

Draw a picture for the multiplication

$$(x + 2) * (x + 4).$$

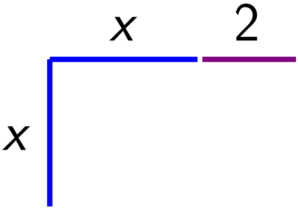
Multiplication $(x+2)(x+4)$

Multiplication $(x+2)(x+4)$

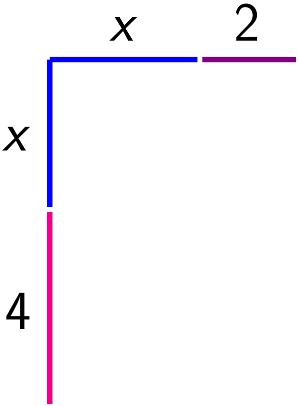
Multiplication $(x+2)(x+4)$

$x \quad 2$

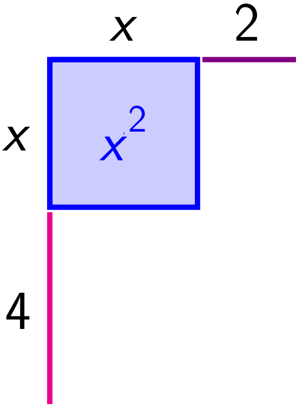
Multiplication $(x+2)(x+4)$



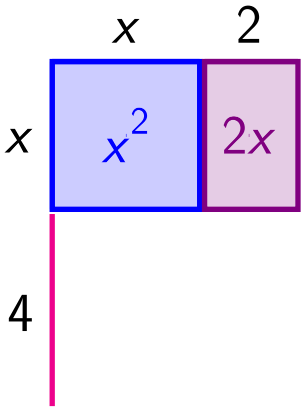
Multiplication $(x+2)(x+4)$



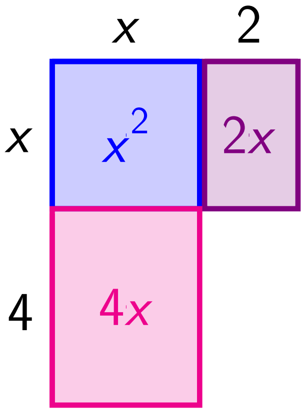
Multiplication $(x + 2)(x + 4)$



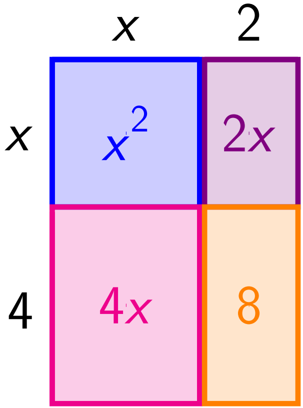
Multiplication $(x+2)(x+4)$



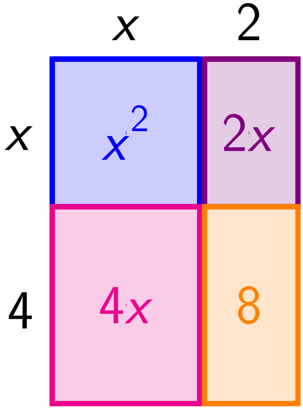
Multiplication $(x+2)(x+4)$



Multiplication $(x + 2)(x + 4)$



Multiplication $(x + 2)(x + 4)$



This gives the area $x^2 + 2x + 4x + 4 * 2$, or $x^2 + 6x + 8$, which is what the distributive property would give.

Factoring is the reverse process of multiplication.

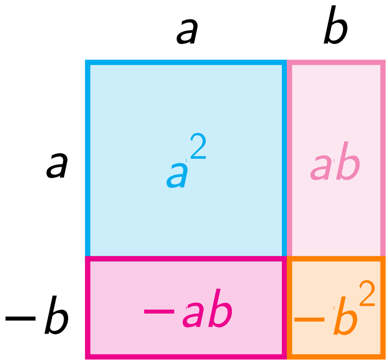
Special Types of Factoring

Draw a picture for the following

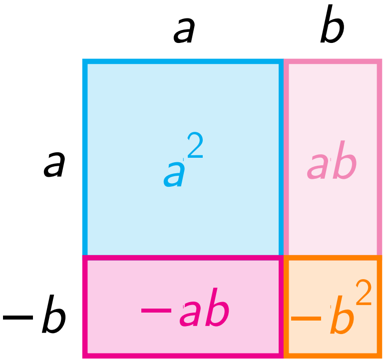
$$(a + b)(a - b)$$

What happened?

Multiplication $(a + b)(a - b)$



Multiplication $(a+b)(a-b)$



This gives the area $a^2 + ab + -ab + -b^2$. Notice that the area of ab is the same as $-ab$, but they cancel because they have opposite signs. (In general negative area does not make sense.)

$$(a + b)(a - b)$$

$$(a+b)(a-b)$$

$$= a^2 + ab + -ab + -b^2$$

$$(a+b)(a-b)$$

$$= a^2 + ab + -ab + -b^2$$

$$= a^2 + \cancel{ab} \cancel{-ab} - b^2$$

$$(a+b)(a-b)$$

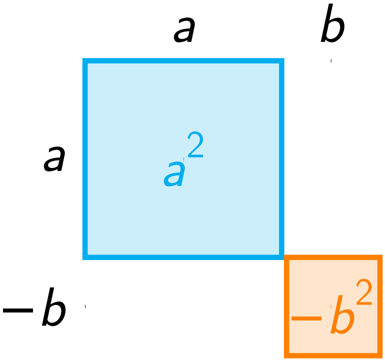
$$= a^2 + ab + -ab + -b^2$$

$$= a^2 + \cancel{ab} \cancel{-ab} - b^2$$

$$= \boxed{a^2 - b^2}$$

Geometrically we have the following picture.

Multiplication $(a+b)(a-b)$

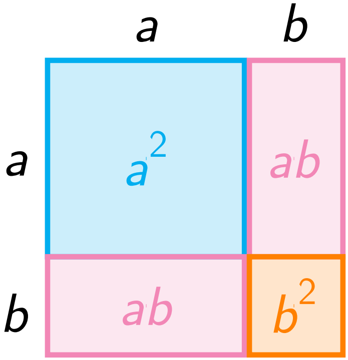


Area $= a^2 - b^2$.

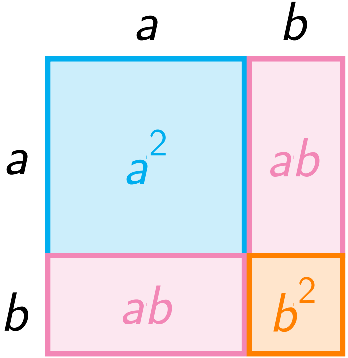
Draw a picture for the following

$$(a+b)(a+b)$$

Multiplication $(a+b)(a+b)$



Multiplication $(a+b)(a+b)$



This gives the area

$a^2 + ab + ab + b^2 = \boxed{a^2 + 2ab + b^2}$.

$$(a + b)(a + b)$$

$$(a+b)(a+b)$$

$$= a^2 + ab + ab + b^2$$

$$(a+b)(a+b)$$

$$= a^2 + ab + ab + b^2$$

$$= \boxed{a^2 + 2ab + b^2}$$

Factor the following expressions

$$25 - y^2$$

Factor the following expressions

$$25 - y^2$$

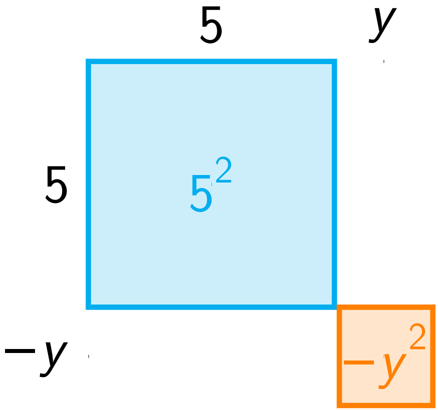
$$= 5^2 - y^2$$

Factor the following expressions

$$25 - y^2$$

$$= 5^2 - y^2$$

$$= \boxed{(5+y)(5-y)}$$



Area $= 25 - y^2$

Factor the following expressions

$$9z^2 + 64$$

Factor the following expressions

$$9z^2 + 64$$

9 and 64 are relatively prime.

Factor the following expressions

$$9x^2 - 4y^2$$

Factor the following expressions

$$9x^2 - 4y^2$$

$$= 3^2x^2 - 2^2y^2$$

Factor the following expressions

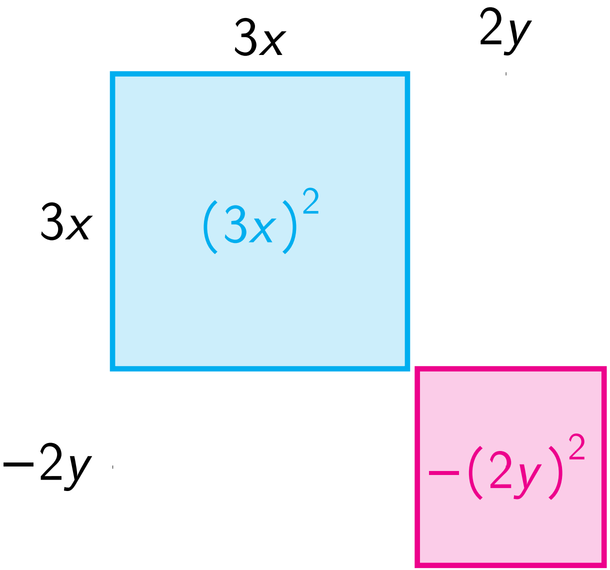
$$9x^2 - 4y^2$$

$$= 3^2x^2 - 2^2y^2$$

$$= (3x)^2 - (2y)^2$$

Factor the following expressions

$$
\begin{aligned}
& 9x^2 - 4y^2 \\
&= 3^2x^2 - 2^2y^2 \\
&= (3x)^2 - (2y)^2 \\
&= \boxed{(3x+2y)(3x-2y)}
\end{aligned}
$$



$$\text{Area} = 9x^2 - 4y^2$$

Factor the following expressions

$$x^2 - 16$$

Factor the following expressions

$$x^2 - 16$$

$$= x^2 - 4^2$$

Factor the following expressions

$$x^2 - 16$$

$$= x^2 - 4^2$$

$$= (x + 4)(x - 4)$$

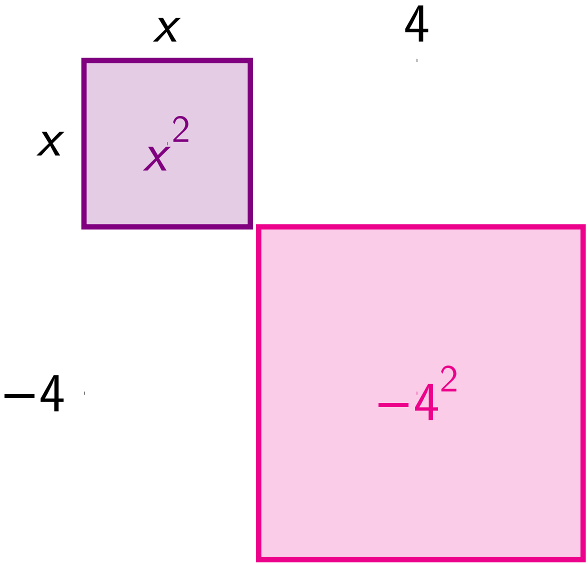
Factor the following expressions

$$x^2 - 16$$

$$= x^2 - 4^2$$

$$= (x+4)(x-4)$$

$$= \boxed{(x+4)(x-4)}$$



Area $= x^2 - 16$

Factor the following expressions

$$100x^3 - x$$

Factor the following expressions

$$100x^3 - x$$

$$= x(100x^2 - 1)$$

Factor the following expressions

$$100x^3 - x$$

$$= x(100x^2 - 1)$$

$$= x(10^2x^2 - 1^2)$$

Factor the following expressions

$$100x^3 - x$$

$$= x(100x^2 - 1)$$

$$= x(10^2x^2 - 1^2)$$

$$= x((10x)^2 - (1)^2)$$

Factor the following expressions

$$
\begin{aligned}
& 100x^3 - x \\
&= x(100x^2 - 1) \\
&= x(10^2x^2 - 1^2) \\
&= x((10x)^2 - (1)^2) \\
&= \boxed{x(10x + 1)(10x - 1)}
\end{aligned}
$$

Factor the following expressions

$$64z^4 - 49z^2$$

Factor the following expressions

$$64z^4 - 49z^2$$

$$= z^2(64z^2 - 49)$$

Factor the following expressions

$$64z^4 - 49z^2$$

$$= z^2(64z^2 - 49)$$

$$= z^2(8^2z^2 - 7^2)$$

Factor the following expressions

$$64z^4 - 49z^2$$

$$= z^2(64z^2 - 49)$$

$$= z^2(8^2z^2 - 7^2)$$

$$= z^2((8z)^2 - (7)^2)$$

Factor the following expressions

$$
\begin{aligned}
&64z^4 - 49z^2 \\
&= z^2(64z^2 - 49) \\
&= z^2(8^2z^2 - 7^2) \\
&= z^2((8z)^2 - (7)^2) \\
&= \boxed{z^2(8z + 7)(8z - 7)}
\end{aligned}
$$

Factor the following expressions

$$9z^2 - 24z + 16$$

Factor the following expressions

$$9z^2 - 24z + 16$$

$$3^2z^2 - (2)(12)z + 4^2$$

Factor the following expressions

$$9z^2 - 24z + 16$$

$$3^2z^2 - (2)(12)z + 4^2$$

$$= (3z - 4)(3z - 4)$$

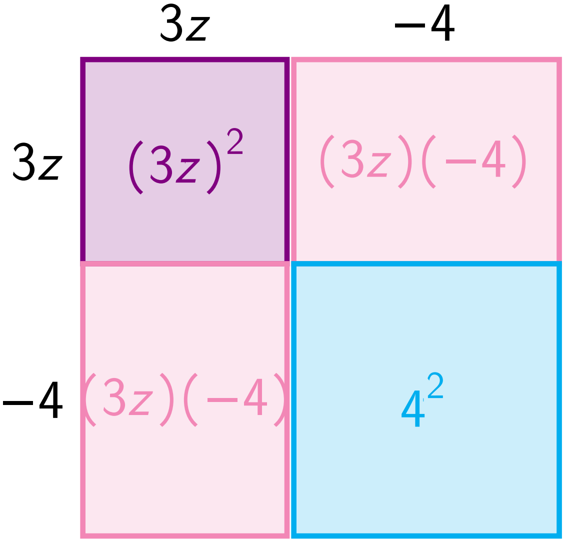
Factor the following expressions

$$9z^2 - 24z + 16$$

$$3^2z^2 - (2)(12)z + 4^2$$

$$= (3z-4)(3z-4)$$

$$= \boxed{(3z-4)^2}$$



$$\text{Area} = 9z^2 + -12z + -12z + 16$$

Factor the following expressions

$$a^2b^2 - c^2d^2$$

Factor the following expressions

$$a^2b^2 - c^2d^2$$

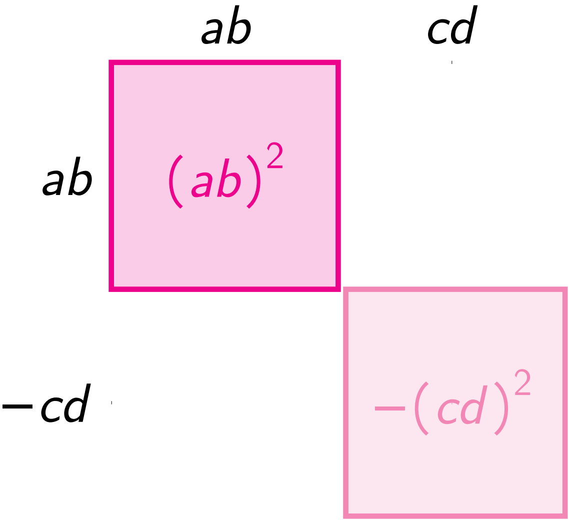
$$= (ab)^2 - (cd)^2$$

Factor the following expressions

$$a^2b^2 - c^2d^2$$

$$= (ab)^2 - (cd)^2$$

$$\boxed{(ab + cd)(ab - cd)}$$



$$\text{Area} = a^2b^2 - c^2d^2$$

Factor the following expressions

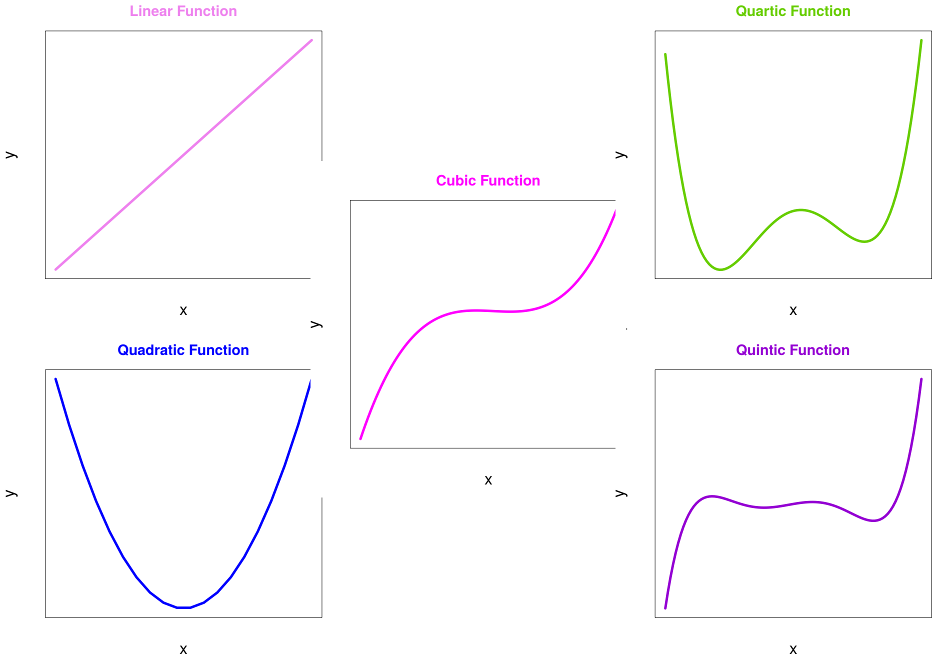
$$x^2 + 25$$

Factor the following expressions

$$x^2 + 25$$

1 and 25 are relatively prime.

Polynomial Equations


Linear Function
Quartic Function
Cubic Function
Quadratic Function
Quintic Function
x
y

Solve the following Polynomial Equation

$$4x^2 - 25 = 0$$

Solve the following Polynomial Equation

$$4x^2 - 25 = 0$$

$$(2x)^2 - 5^2 = 0$$

Solve the following Polynomial Equation

$$4x^2 - 25 = 0$$

$$(2x)^2 - 5^2 = 0$$

$$(2x + 5)(2x - 5) = 0$$

Solve the following Polynomial Equation

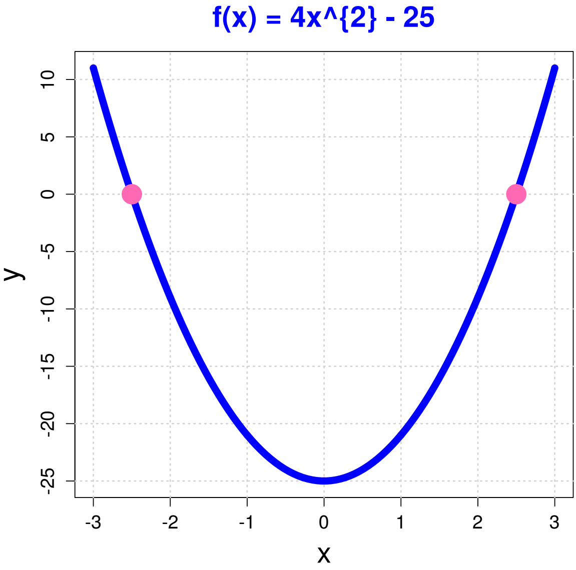
$$4x^2 - 25 = 0$$

$$(2x)^2 - 5^2 = 0$$

$$(2x+5)(2x-5) = 0$$

$$\boxed{x = \frac{-5}{2} \text{ or } x = \frac{5}{2}}$$

Roots of Quadratic Polynomial



Solve the following Polynomial Equation

$$9x^2 + 1 = 6x$$

Solve the following Polynomial Equation

$$9x^2 + 1 = 6x$$

$$9x^2 - 6x + 1 = 0$$

Solve the following Polynomial Equation

$$9x^2 + 1 = 6x$$

$$9x^2 - 6x + 1 = 0$$

$$(3x - 1)(3x - 1) = 0$$

Solve the following Polynomial Equation

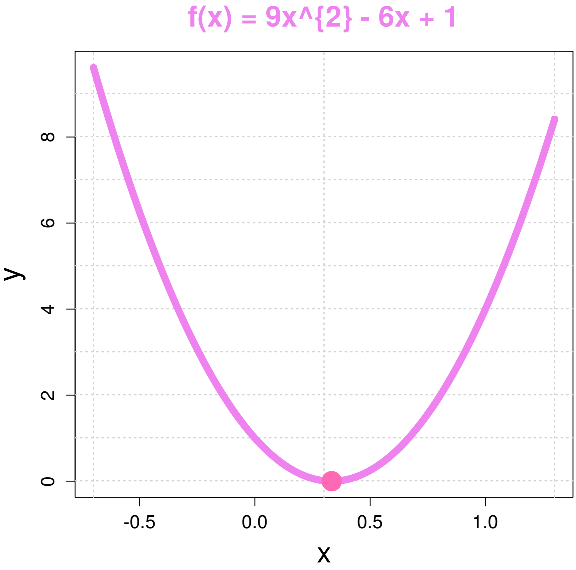
$$9x^2 + 1 = 6x$$

$$9x^2 - 6x + 1 = 0$$

$$(3x - 1)(3x - 1) = 0$$

$$\boxed{x = \frac{1}{3}}$$

Root of Quadratic Polynomial



Solve the following Polynomial Equation

$$4x^2 = 64$$

Solve the following Polynomial Equation

$$4x^2 = 64$$

$$4x^2 - 64 = 0$$

Solve the following Polynomial Equation

$$4x^2 = 64$$

$$4x^2 - 64 = 0$$

$$4(x^2 - 16) = 0$$

Solve the following Polynomial Equation

$$4x^2 = 64$$

$$4x^2 - 64 = 0$$

$$4(x^2 - 16) = 0$$

$$4(x^2 - 4^2) = 0$$

Solve the following Polynomial Equation

$$4x^2 = 64$$

$$4x^2 - 64 = 0$$

$$4(x^2 - 16) = 0$$

$$4(x^2 - 4^2) = 0$$

$$4(x + 4)(x - 4) = 0$$

Solve the following Polynomial Equation

$$4x^2 = 64$$

$$4x^2 - 64 = 0$$

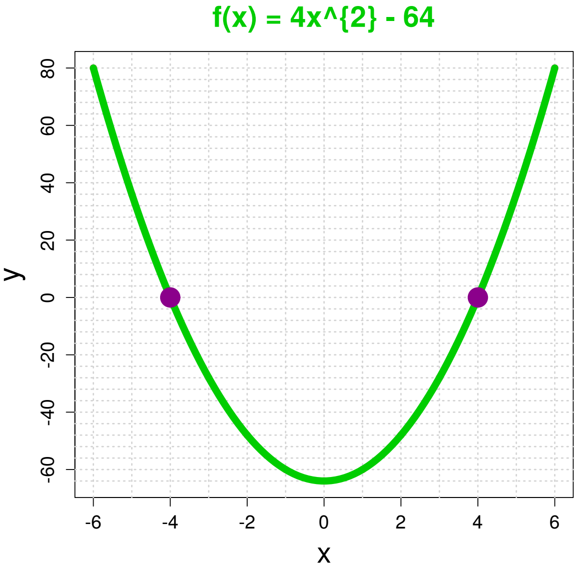
$$4(x^2 - 16) = 0$$

$$4(x^2 - 4^2) = 0$$

$$4(x + 4)(x - 4) = 0$$

$$\boxed{x = -4 \text{ or } x = 4}$$

Roots of Quadratic Polynomial



Solve the following Polynomial Equation

$$x^3 = 16x$$

Solve the following Polynomial Equation

$$x^3 = 16x$$

$$x^3 - 16x = 0$$

Solve the following Polynomial Equation

$$x^3 = 16x$$

$$x^3 - 16x = 0$$

$$x(x^2 - 16) = 0$$

Solve the following Polynomial Equation

$$x^3 = 16x$$

$$x^3 - 16x = 0$$

$$x(x^2 - 16) = 0$$

$$x(x^2 - 4^2) = 0$$

Solve the following Polynomial Equation

$$x^3 = 16x$$

$$x^3 - 16x = 0$$

$$x(x^2 - 16) = 0$$

$$x(x^2 - 4^2) = 0$$

$$x(x + 4)(x - 4) = 0$$

Solve the following Polynomial Equation

$$x^3 = 16x$$

$$x^3 - 16x = 0$$

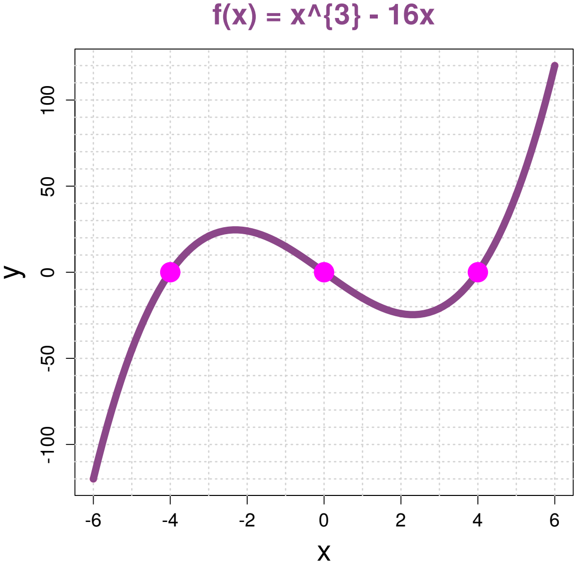
$$x(x^2 - 16) = 0$$

$$x(x^2 - 4^2) = 0$$

$$x(x + 4)(x - 4) = 0$$

$$\boxed{x = 0 \text{ or } x = -4 \text{ or } x = 4}$$

Roots of Quadratic Polynomial



Solve the following Polynomial Equation

$$x^3 + 5x^2 = 9x + 45$$

Solve the following Polynomial Equation

$$x^3 + 5x^2 = 9x + 45$$

$$x^3 + 5x^2 - 9x - 45 = 0$$

Solve the following Polynomial Equation

$$x^3 + 5x^2 = 9x + 45$$

$$x^3 + 5x^2 - 9x - 45 = 0$$

$$\underbrace{x^3 + 5x^2} - \underbrace{9x - 45} = 0$$

Solve the following Polynomial Equation

$$x^3 + 5x^2 = 9x + 45$$

$$x^3 + 5x^2 - 9x - 45 = 0$$

$$\underbrace{x^3 + 5x^2} - \underbrace{9x - 45} = 0$$

$$\underbrace{x^2(x + 5)} - \underbrace{9(x + 5)} = 0$$

Solve the following Polynomial Equation

$$x^3 + 5x^2 = 9x + 45$$

$$x^3 + 5x^2 - 9x - 45 = 0$$

$$\underbrace{x^3 + 5x^2} - \underbrace{9x - 45} = 0$$

$$\underbrace{x^2(x + 5)} - \underbrace{9(x + 5)} = 0$$

$$(x^2 - 9)(x + 5) = 0$$

Solve the following Polynomial Equation

$$(x^2 - 9)(x + 5) = 0$$

Solve the following Polynomial Equation

$$(x^2 - 9)(x + 5) = 0$$

$$(x^2 - 3^2)(x + 5) = 0$$

Solve the following Polynomial Equation

$$(x^2 - 9)(x + 5) = 0$$

$$(x^2 - 3^2)(x + 5) = 0$$

$$(x + 3)(x - 3)(x + 5) = 0$$

Solve the following Polynomial Equation

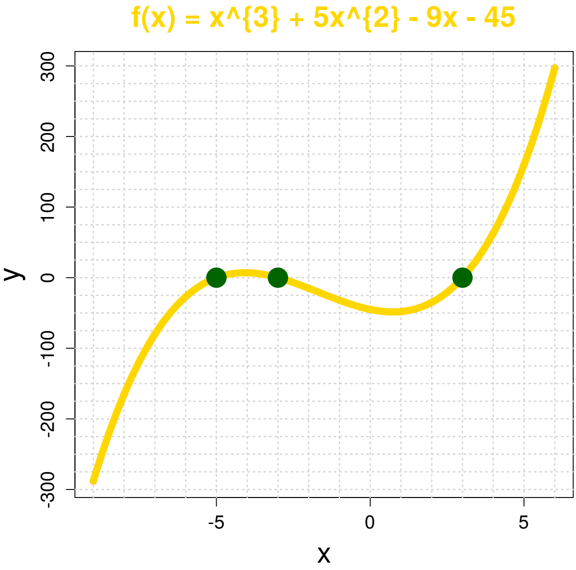
$$(x^2 - 9)(x + 5) = 0$$

$$(x^2 - 3^2)(x + 5) = 0$$

$$(x + 3)(x - 3)(x + 5) = 0$$

$$\boxed{x = -3 \text{ or } x = 3 \text{ or } x = -5}$$

Roots of Quadratic Polynomial



Next Time

Domain and Range of Rational

Functions

& Rational Expressions