Algebra Lecture 13

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Topics

Topics for Today

- Polynomial Functions & Models of real data
- Factoring

Recall what is a function.

A relation in which each x-coordinate is matched with only one y-coordinate is said to describe y as a function of x.

What test is used to determine if a relation is a function?

Vertical Line Test

What is a Polynomial Function?

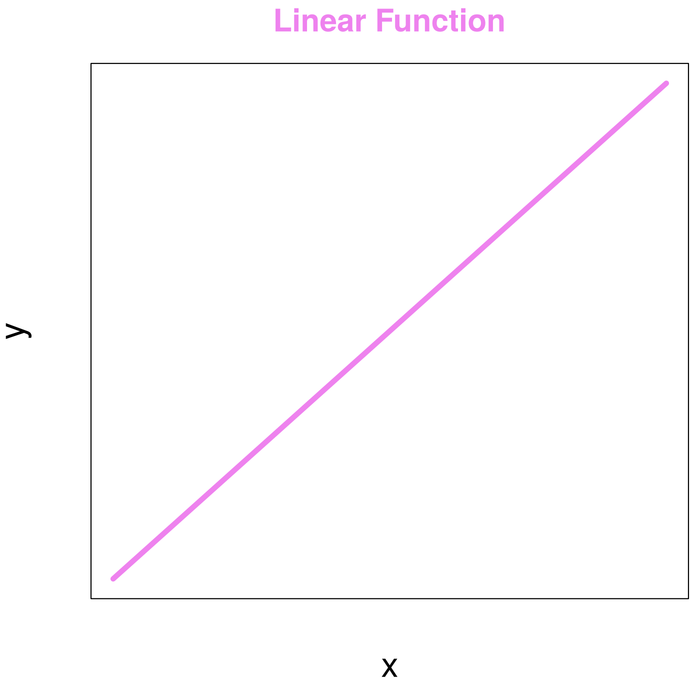
A Polynomial Function is a function of the form

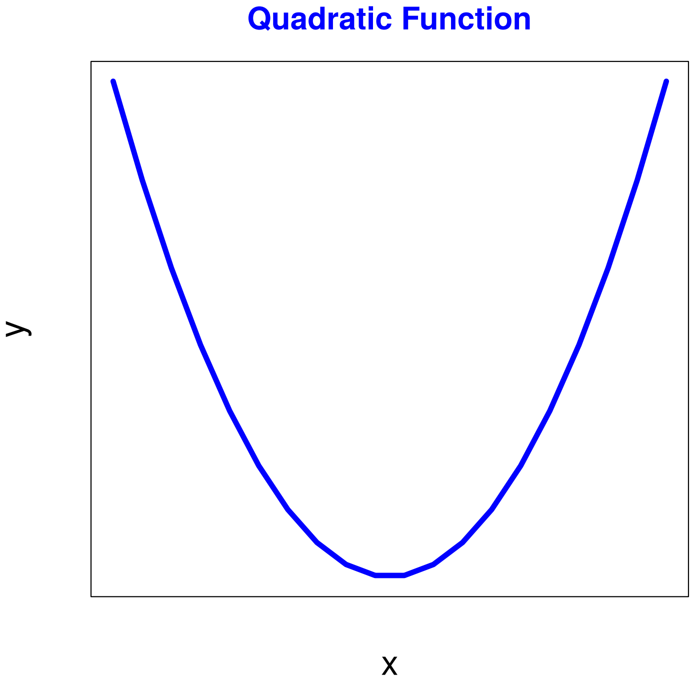
$$y = f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x^1 + a_0$$

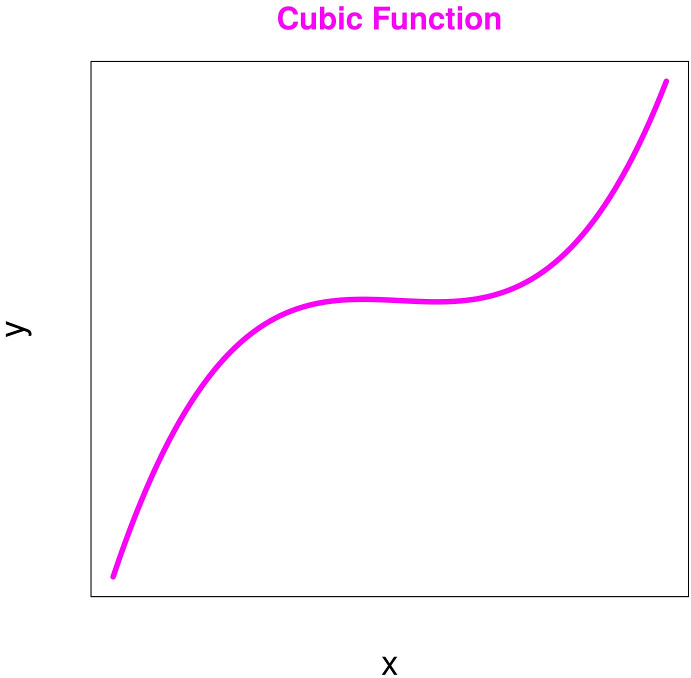
Examples

Types of Polynomial Functions

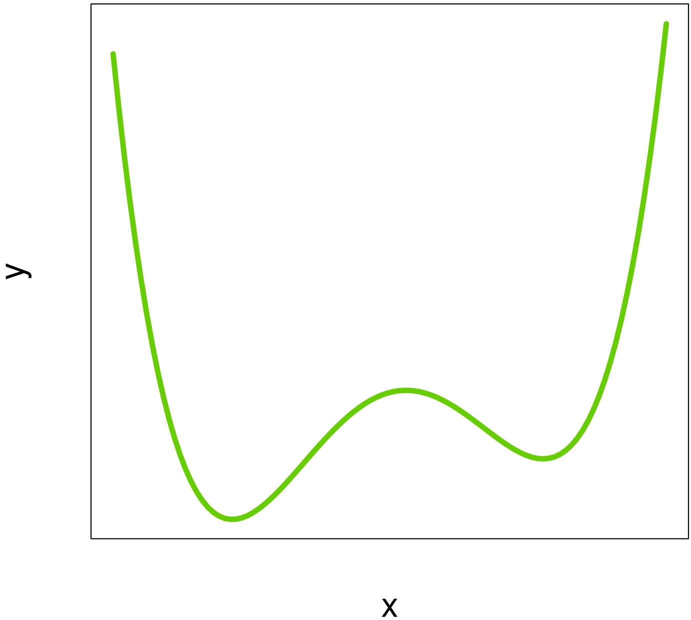
$y = a_0$	Constant Function
$y = a_1 x + a_0$	Linear Function
$y = a_2 x^2 + a_1 x + a_0$	Quadratic Functio
$y = a_3 x^3 + a_2 x^2 + a_1 x + a_0$	Cubic Function
$y = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$	Quartic Function


Linear Function
y
x

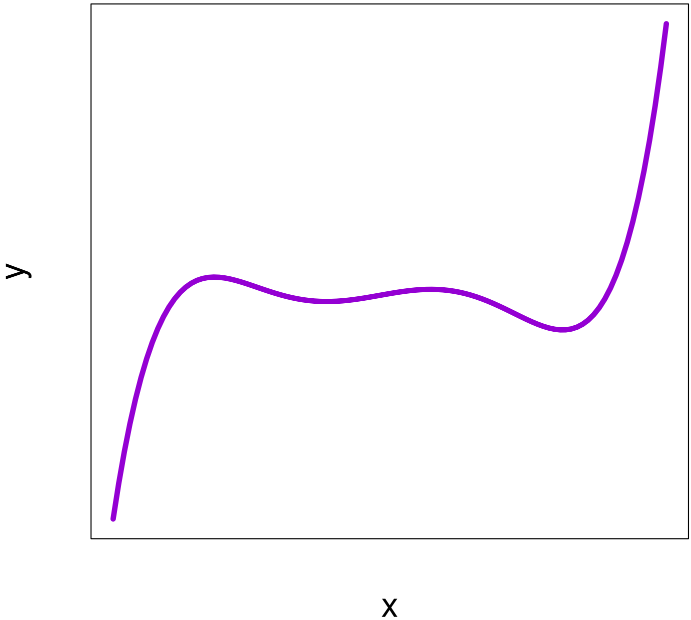

Quadratic Function
y
x


Cubic Function
y
x

Quartic Function



Quintic Function



Suppose we have the following cubic function

$$f(x) = -x^3 - 5x + 6$$

Suppose we have the following cubic function

$$f(x) = -x^3 - 5x + 6$$

Find the value for $f(-2)$.

$f(x) = -x^3 - 5x + 6$

Substitute -2 for x

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Substitute -2 for x

$$f(-2) = -(-2)^3 - 5(-2) + 6$$

$f(x) = -x^3 - 5x + 6$

Substitute -2 for x

$$f(-2) = -(-2)^3 - 5(-2) + 6$$

$$= -(-8) - 5(-2) + 6$$

$f(x) = -x^3 - 5x + 6$

Substitute -2 for x

$$f(-2) = -(-2)^3 - 5(-2) + 6$$

$$= -(-8) - 5(-2) + 6$$

$$= -(-8) + 10 + 6$$

$f(x) = -x^3 - 5x + 6$

Substitute -2 for x

$$f(-2) = -(-2)^3 - 5(-2) + 6$$

$$= -(-8) - 5(-2) + 6$$

$$= -(-8) + 10 + 6$$

$$= 8 + 10 + 6$$

$f(x) = -x^3 - 5x + 6$

Substitute -2 for x

$$f(-2) = -(-2)^3 - 5(-2) + 6$$

$$= -(-8) - 5(-2) + 6$$

$$= -(-8) + 10 + 6$$

$$= 8 + 10 + 6$$

$$= 24$$

$f(x) = -x^3 - 5x + 6$

Substitute -2 for x

$$f(-2) = -(-2)^3 - 5(-2) + 6$$

$$= -(-8) - 5(-2) + 6$$

$$= -(-8) + 10 + 6$$

$$= 8 + 10 + 6$$

$$= 24$$

$$\boxed{f(-2) = 24}$$

Modelling Real World Data

The following polynomial function

$$H(t) = 1.875t^2 - 30t + 200$$

models a typical athlete's heart (♡) rate in beats per minute after exercise has stopped, where $0 \le t \le 8$.

What is the initial heart rate when the athlete stops exercising?

In other words, what is the heart rate at the time $t = 0$?

Heart (♡) rate $H(t) = 1.875t^2 - 30t + 200, 0 \leq t \leq 8$

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$$H(0) = 1.875(0)^2 - 30(0) + 200$$

$$= 1.875(0) - 0 + 200$$

Heart (♡) rate $H(t) = 1.875t^2 - 30t + 200, 0 \le t \le 8$

$$H(0) = 1.875(0)^2 - 30(0) + 200$$

$$= 1.875(0) - 0 + 200$$

$$= 0 - 0 + 200$$

Heart (♡) rate $H(t) = 1.875t^2 - 30t + 200, 0 \leq t \leq 8$

$$H(0) = 1.875(0)^2 - 30(0) + 200$$

$$= 1.875(0) - 0 + 200$$

$$= 0 - 0 + 200$$

$$\boxed{H(0) = 200}$$

What is the heart rate after eight minutes?

In other words, what is the heart rate at the time $t = 8$?

Heart (♡) rate $H(t) = 1.875t^2 - 30t + 200, 0 \leq t \leq 8$

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$$H(8) = 1.875(8)^2 - 30(8) + 200$$

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$$H(8) = 1.875(8)^2 - 30(8) + 200$$

$$= 1.875(64) - 240 + 200$$

Heart (♡) rate $H(t) = 1.875t^2 - 30t + 200, 0 \le t \le 8$

$$H(8) = 1.875(8)^2 - 30(8) + 200$$

$$= 1.875(64) - 240 + 200$$

$$= 120 - 40$$

Heart (♡) rate $H(t) = 1.875t^2 - 30t + 200, 0 \le t \le 8$

$$H(8) = 1.875(8)^2 - 30(8) + 200$$

$$= 1.875(64) - 240 + 200$$

$$= 120 - 40$$

$$\boxed{H(8) = 80}$$

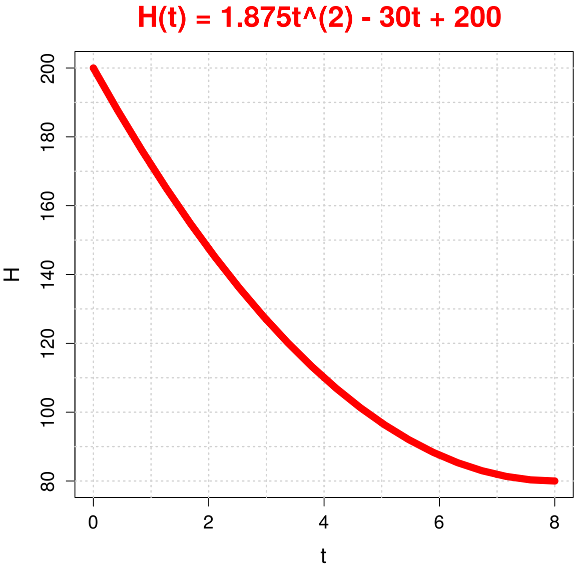
What kind of a polynomial function is the heart (♡) rate model?

$$H(t) = 1.875t^2 - 30t + 200$$

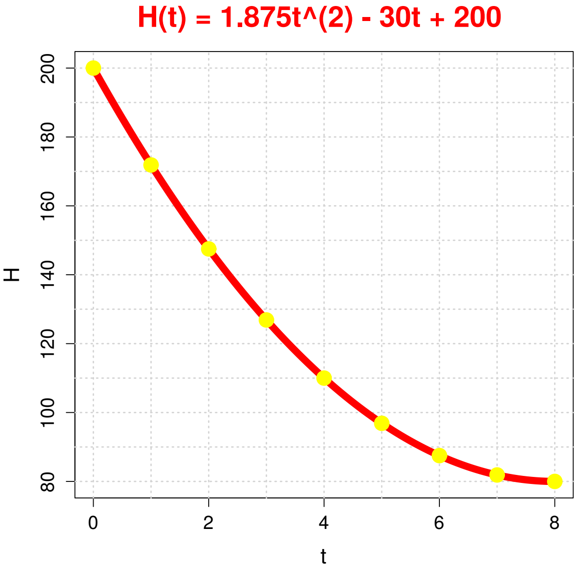
The heart (♡) rate model is a

Quadratic Polynomial Function.

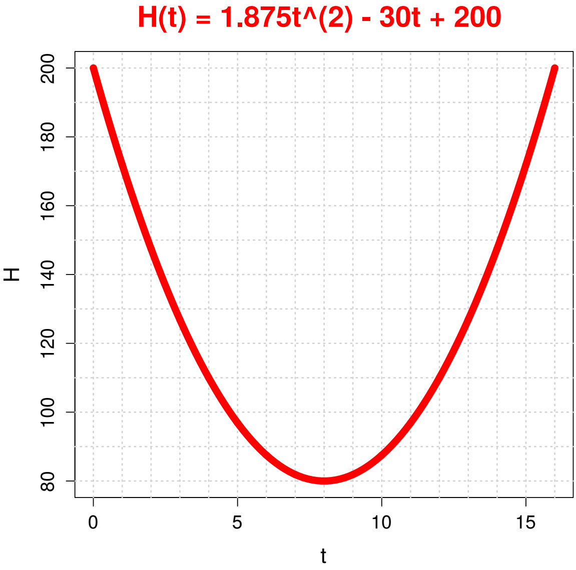
$H(t) = 1.875t^2 - 30t + 200$ is a Quadratic Polynomial



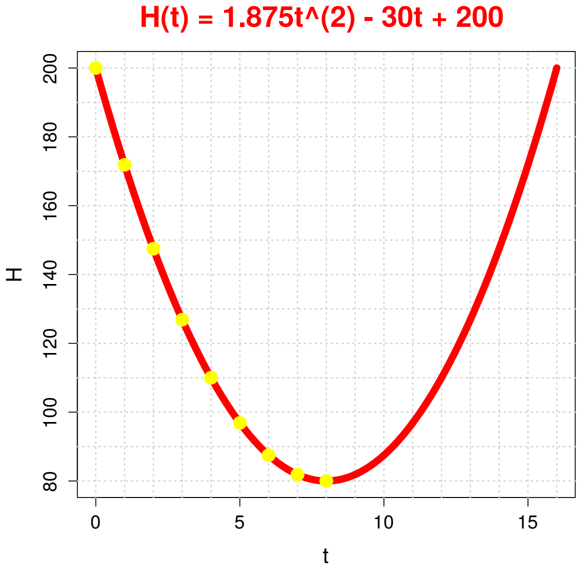
$H(t) = 1.875t^2 - 30t + 200$ is a Quadratic Polynomial



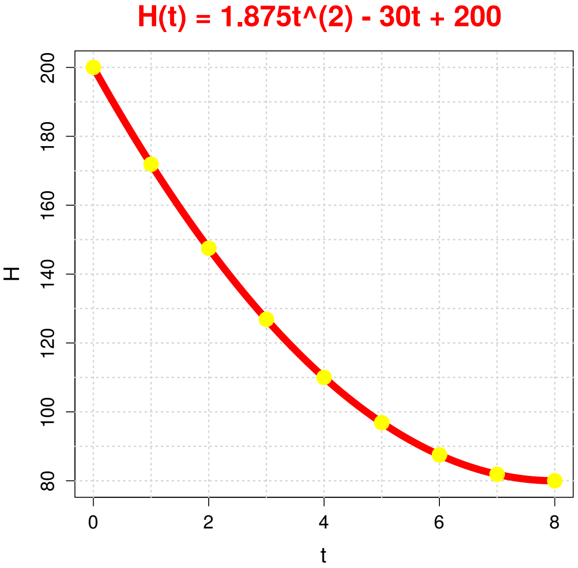
$H(t) = 1.875t^2 - 30t + 200$ is a Quadratic Polynomial



$H(t) = 1.875t^2 - 30t + 200$ is a Quadratic Polynomial



Create a table for the heart model $H(t)$. Begin the table at time three minutes and increment by one minute. At what time was the athlete's heart rate between 80 and 110 beats per minute, inclusive?



H(t) = 1.875t^(2) - 30t + 200
H
200
180
160
140
120
100
80
0
2
4
6
8
t

The polynomial function

$$f(x) = -0.064x^3 + 0.56x^2 + 2.9x + 61$$

models the ocean temperature in degrees Fahrenheit at Naples, Florida. Here $x = 1$ corresponds to the month of January, $x = 2$ corresponds to February, and so on.

What is the ocean temperature in March?

In other words, what is the ocean temperature at time $x = 3$?

$$f(x) = -0.064x^3 + 0.56x^2 + 2.9x + 61$$

$$f(x) = -0.064x^3 + 0.56x^2 + 2.9x + 61$$

$$f(3) = -0.064(3)^3 + 0.56(3)^2 + 2.9(3) + 61$$

$$f(x) = -0.064x^3 + 0.56x^2 + 2.9x + 61$$

$$f(3) = -0.064(3)^3 + 0.56(3)^2 + 2.9(3) + 61$$

$$\boxed{73.012^\circ \sim 73^\circ}$$

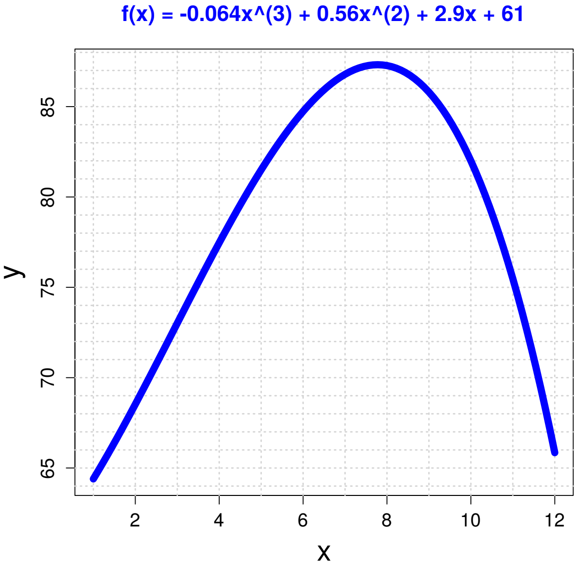
What kind of a polynomial function is the Ocean Temperature Model?

$$f(x) = -0.064x^3 + 0.56x^2 + 2.9x + 61$$

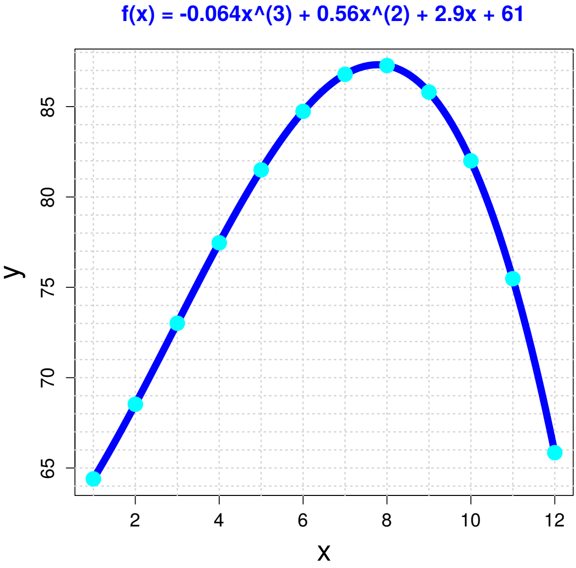
The Ocean Temperature Model is a

Cubic Polynomial Function.

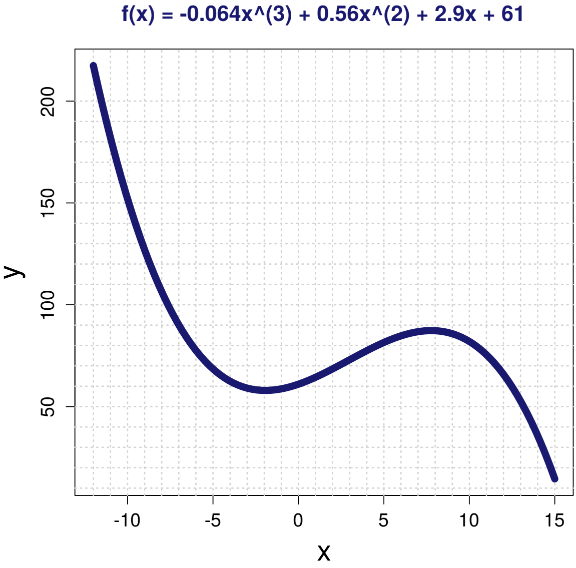
Ocean Temperature Model Cubic Polynomial



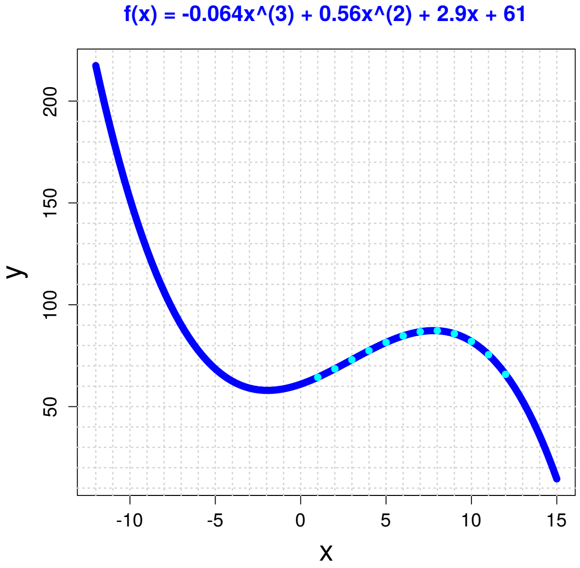
Ocean Temperature Model Cubic Polynomial



Ocean Temperature Model Cubic Polynomial



Ocean Temperature Model Cubic Polynomial



Factoring Polynomials

Factor the following expression:

$$24m^2n^3 - 36m^3n^2$$

Factor the following expression:

$$24m^2n^3 - 36m^3n^2$$

$$= \boxed{12m^2n^2(2n - 3m)}$$

Factor the following expression:

$$-8z^5 - 24z^4$$

Factor the following expression:

$$-8z^5 - 24z^4$$

$$= \boxed{-8z^4(z + 3)}$$

Factor the following expression:

$$5(x - 1) - 2x(x - 1)$$

Factor the following expression:

$$5(x-1)-2x(x-1)$$

$$=\boxed{(x-1)(5-2x)}$$

Factor the following expression:

$$4x^3 + 3x^2 + 8x + 6$$

Factor the following expression:

$$4x^3 + 3x^2 + 8x + 6$$

$$= \underbrace{4x^3 + 3x^2} + \underbrace{8x + 6}$$

Factor the following expression:

$$4x^3 + 3x^2 + 8x + 6$$

$$= \underbrace{4x^3 + 3x^2} + \underbrace{8x + 6}$$

$$= x^2(4x + 3) + 2(4x + 3)$$

Factor the following expression:

$$4x^3 + 3x^2 + 8x + 6$$

$$= \underbrace{4x^3 + 3x^2} + \underbrace{8x + 6}$$

$$= x^2(4x + 3) + 2(4x + 3)$$

$$= \boxed{(4x + 3)(x^2 + 2)}$$

Factor the following expression:

$$6x^3 - 15x^2 - 4x + 10$$

Factor the following expression:

$$6x^3 - 15x^2 - 4x + 10$$

$$= \underbrace{6x^3 - 15x^2} + \underbrace{-4x + 10}$$

Factor the following expression:

$$6x^3 - 15x^2 - 4x + 10$$

$$= \underbrace{6x^3 - 15x^2} + \underbrace{-4x + 10}$$

$$= 3x^2(2x - 5) - 2(2x - 5)$$

Factor the following expression:

$$6x^3 - 15x^2 - 4x + 10$$

$$= \underbrace{6x^3 - 15x^2} + \underbrace{-4x + 10}$$

$$= 3x^2(2x - 5) - 2(2x - 5)$$

$$= \boxed{(2x - 5)(3x^2 - 2)}$$

Factor the following expression:

$$ax + bx - ay - by$$

Factor the following expression:

$$ax + bx - ay - by$$

$$= \underbrace{ax + bx} + \underbrace{-ay - by}$$

Factor the following expression:

$$ax + bx - ay - by$$

$$= \underbrace{ax + bx} + \underbrace{-ay - by}$$

$$= x(a + b) - y(a + b)$$

Factor the following expression:

$$ax + bx - ay - by$$

$$= \underbrace{ax + bx} + \underbrace{-ay - by}$$

$$= x(a + b) - y(a + b)$$

$$= \boxed{(a + b)(x - y)}$$

Solve for x.

$$ax + bx = c$$

Solve for x.

$$ax + bx = c$$

$$x(a + b) = c$$

Solve for x.

$$ax + bx = c$$

$$x(a + b) = c$$

$$\boxed{x = \frac{c}{(a + b)}}$$

Solve for y.

$$ay - de = by$$

Solve for y.

$$ay - de = by$$

$$ay - by = de$$

Solve for y.

$$ay - de = by$$

$$ay - by = de$$

$$y(a - b) = de$$

Solve for y.

$$ay - de = by$$

$$ay - by = de$$

$$y(a - b) = de$$

$$\boxed{y = \frac{de}{(a - b)}}$$

Solving Equations

Solve the following equation:

$$abc = 0$$

Solve the following equation:

$$abc = 0$$

$a = 0$ or

Solve the following equation:

$$abc = 0$$

$$a = 0 \text{ or } b = 0 \text{ or}$$

Solve the following equation:

$$abc = 0$$

$$a = 0 \text{ or } b = 0 \text{ or } c = 0$$

Solve the following equation:

$$4x(3x - 7) = 0$$

Solve the following equation:

$$4x(3x - 7) = 0$$

$$\boxed{\{0, 7/3\}}$$

Solve the following equation:

$$7m(3n+1)=0$$

Solve the following equation:

$$7m(3n+1) = 0$$

$$\boxed{m = 0, n = \frac{-1}{3}}$$

Solve the following equation:

$$(3x - 5)(7x + 11) = 0$$

Solve the following equation:

$$(3x - 5)(7x + 11) = 0$$

$$\boxed{\{5/3, -11/7\}}$$

Solve the following equation:

$$7x^2 - x = 0$$

Solve the following equation:

$$7x^2 - x = 0$$

$$x(7x - 1) = 0$$

Solve the following equation:

$$7x^2 - x = 0$$

$$x(7x - 1) = 0$$

$$\boxed{\{0, 1/7\}}$$

Solve the following equation:

$$15x^2 = 5x$$

Solve the following equation:

$$15x^2 = 5x$$

$$15x^2 - 5x = 0$$

Solve the following equation:

$$15x^2 = 5x$$

$$15x^2 - 5x = 0$$

$$5x(3x - 1) = 0$$

Solve the following equation:

$$15x^2 = 5x$$

$$15x^2 - 5x = 0$$

$$5x(3x - 1) = 0$$

$$\boxed{\{0, 1/3\}}$$

Solve the following equation:

$$51x = 34x^2$$

Solve the following equation:

$$51x = 34x^2$$

$$0 = 34x^2 - 51x$$

Solve the following equation:

$$51x = 34x^2$$

$$0 = 34x^2 - 51x$$

$$0 = 17x(2x - 3)$$

Solve the following equation:

$$51x = 34x^2$$

$$0 = 34x^2 - 51x$$

$$0 = 17x(2x - 3)$$

$$\boxed{\{0, 3/2\}}$$

Solve the following equation:

$$45x^4 - 30x^3 = 0$$

Solve the following equation:

$$45x^4 - 30x^3 = 0$$

$$15x^3(3x - 2) = 0$$

Solve the following equation:

$$45x^4 - 30x^3 = 0$$

$$15x^3(3x - 2) = 0$$

$$\boxed{\{0, 2/3\}}$$

Next Time

Factoring Trinomials & Special Cases

of Factoring