$\boxed{1.1.16}$ Show that there are no retractions $r: X \to A$ in the following cases:

(a) $X = \mathbb{R}^3$ with A any subspace homeomorphic to S^1.

Suppose $\exists\ r: X \to A$

$$A \xhookrightarrow{i} X \xrightarrow{r} A$$

$$ri = \mathbb{1}_A$$

$$\underbrace{\overset{\text{nontrivial}}{\pi_1(A,x_0)} \xhookrightarrow{i_*} \overset{\text{trivial}}{\pi_1(X,x_0)} \xrightarrow{r_*} \overset{\text{nontrivial}}{\pi_1(A,x_0)}}_{r_* i_*}$$

$$\boxed{\begin{aligned} (ri)_* &= r_* i_* \\ (\mathbb{1}_A)_* &= \mathbb{1}_A \end{aligned}} \qquad \boxed{\begin{aligned} &i_* \quad 1\text{-}1 \\ &r_* \quad \text{onto} \end{aligned}}$$

$X = \mathbb{R}^3$ is simply connected since it is path-connected and every loop is path homotopic to the constant loop by the straight-line homotopy.

$\Longrightarrow \pi_1(X,x_0) = 1$ is trivial

Since $\pi_1(A,x_0)$ is nontrivial we have a contradiction. ↯